COMPETITIVE NURSE ROSTERING AND REROSTERING

By

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A Dissertation Presented in Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

ARIZONA STATE UNIVERSITY

May 2008

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **01 MAY 2008**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **Competitive Nurse Rostering and Rerostering**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Arizona State University**	8. PERFORMING ORGANIZATION REPORT NUMBER **CIO8-0005**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **The Department of the Air Force AFIT/ENEL, Bldg 162275 D Street WPAFB, OH 45433**	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**153**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

COMPETITIVE NURSE ROSTERING AND REROSTERING

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January 2008

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ABSTRACT

Nurse rostering is the assignment of specific nurses to specific shifts for a future scheduling period. The work schedule that is created is called a roster. The reconstruction of a disrupted roster is called rerostering. When solving the rostering and rerostering problems there are two considerations: the organization's costs and the nurses' preferences. Traditional solution methods, often based on integer programs (IP), have two short comings; first, they rely on one objective function to represent both the organization's and nurses' goals; second, rostering requires either the complete resolving of the rostering problem or a new solution method to fix the roster. We propose three agent-based auction heuristics, Competitive Nurse Rostering (CNR), an extension called CNR-Iterated Local Search (CNR-ILS), and an extension of CNR-ILS called CNR-Rerostering (CNRR). These heuristics are the first nurse rostering methods that model each nurse's preferences in separate objective functions. The heuristics are the first competitive agent-based rostering and rerostering methods. They uniquely separate the organizational cost and nurse preference problems by constraining the preference problem's solutions space to alternate cost optimal solutions. CNRR is the first rostering solution that can reroster nurses. When tested in a real hospital, CNR and CNR-ILS solved the rostering problem 99% faster than the hospital's rostering method and an IP solution from the literature. Nurses consistently favored the solutions from CNR-ILS compared to those from CNR, the IP and the hospital. CNRR finds solutions to the rerostering problem over 90% of the time. Less than one sixth of the solutions had a serious impact to nurse preferences.

ACKNOWLEDGEMENTS

I would like to acknowledge those whose help and support directly contributed to the successful completion of this dissertation. My deepest thanks to my wife Laurel Chiaramonte who coauthored Chapter Two and helped setup and sustain my research effort at Mike O'Callaghan Federal Hospital. A special thanks to my committee chairs Dr. Jeffery Cochran and Dr Teresa Wu. Dr Cochran coauthored Chapter Three and provided me with valuable guidance. Dr. Wu helped focus my research and ensure that I included all the required discussions. Thanks to my father, Charles Chiaramonte, who provided a fresh eye to clarify how I was communicating ideas. I would like to thank the rest of my Dissertation Committee, without whom I would not have completed my Ph.D. Finally I would like to extend my gratitude to the nurses of Mike O'Callaghan Federal Hospital's Air Force Medical Surgical Unit and specifically their Officer In Charge, Major Kathy Savell. Their participation in the research process was central to my success.

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CHAPTER 1 NURSE ROSTERING AND REROSTERING – A PRIMER

1.0 OVERVIEW

This Dissertation proposes a set of three related nurse rostering algorithms designed to improve the perception of fairness in nurse scheduling called Competitive Nurse Rostering (CNR), Competitive Nurse Rostering with Iterated Local Search (CNR-ILS), and Competitive Nurse Rostering and Rerostering (CNRR). Traditionally nurse scheduling is comprised of three phases: forecasting overall staff demand, minimal demand covering, and staff rostering. Currently throughout these phases individual nurse preferences are taken into consideration. The CNR algorithms are different, the mimic actual nurse behavior and isolate the preference models that are used by nurses to evaluate their final schedules.

This document is broken down into five chapters. This first chapter discusses the motivation for improving nurse scheduling techniques, presents an overview of the nurse scheduling literature, gives a high level discussion about the CNR series of algorithms, and highlights the original contribution. The second chapter discusses the detailed implementation of the CNR algorithm. The third chapter presents the details of the CNR-ILS algorithm. The fourth chapter details the CNRR algorithm. The second, third and fourth chapters include analysis of each algorithm's performance during experimentation.

1.1 INTRODUCTION

Hospitals spend more capital on personnel than anything other part of the budget (Ozcan 2005). Not surprisingly the reduction of staffing has been a major focus of past scheduling research. The ongoing nurse shortage is forcing this focus to change.

The nursing corps plays an integral role in our healthcare system; the care they provide consumes nearly 25% - 33% of a hospital's budget (Welton 2006). To support this part of the healthcare system we need to provide adequate recruitment and effective retention (Tierney 2003). The three key factors feeding the nurse shortage are the number of nurses leaving the industry, an aging general population requiring more care and, an aging labor pool (Buchan 2006 and Johnson 2006). Implementing better nurse scheduling methods will help mitigate the problems causing the nursing shortage.

The schedules of nurses affect job satisfaction and influence recruitment. Recent surveys have shown that a nurse's work hours and schedules are among the top reasons for job dissatisfaction (McIntosh 2006). Introducing proper scheduling practices can improve turnover and absenteeism rates (Ozcan 2005). This paper outlines a new scheduling technique that focuses on improving the perceptions of fairness in nurse scheduling.

1.2 PROBLEM DEFINITION

Nurse scheduling can be broken down to three distinct phases: the determination of staff demand requirements, the creation of shifts to satisfy demand, and the rostering of nurses to shifts. Each of these steps has a distinct focus.

Before hiring nurses, a hospital needs to forecast the number of patients that will need treatment. Using this forecast, managers can base the demand for nurses upon nurse-patient ratios. Although these ratios are usually derived from hospital policy the issue of staffing numbers has received enough attention that states have begun to legislate nurse-patient ratios. For example California passed Assembly Bill 394 in 1999 mandating

the State Department of Health issue minimum nurse-staffing ratios for hospitals, the first state to do so.

Nurse staffing levels can be modeled in a number of ways using a patient census and patient acuity. An example model uses the current shift to predict what will be required in future shifts (Siferd and Benton 1994). The model is formulated as follows:

$$A_t = R_{t-1} * (A_{t-1} - d_{t-1} D_{t-1}) + b_{t-1} Y_{t-1} \qquad (1.1)$$

Here A is the total acuity of all patients in the unit in terms of the number of nurses needed. R is the rate of acuity change during a period. D is the number of discharges and d is the mean acuity of discharged patients. Y is the number of admissions, and b is the mean acuity of admissions. Thus the acuity of the next period is a function of the acuity of the prior, the acuity of those discharged, the acuity of those admitted and the change in acuity of those remaining.

Once a hospital ward has determined its' nursing demand, shifts must be developed that include enough nurses to cover that demand. Traditionally this was accomplished using various set covering models. A typical set covering model is an integer program that minimizes the cost of overages while meeting manning requirements. The problem can be formulized as follows:

$$\begin{aligned} &Min \sum x_i && (1.2) \\ &st: && \\ &\sum a_{ij} x_i \geq b_j \;\; \forall j && (1.3) \\ &x_i \geq 0, \;\; x_i \in Z && \\ &a_{ij} \in \{0,1\} && \end{aligned}$$

Where x_i is the number of nurses assigned to schedule i, a_{ij} is a binary value indicating whether schedule i includes time period j, and b_j is the demand for nurses during period j.

The final step of the nurse scheduling problem is rostering. Rostering is the assignment of nurses to specific shifts. Preferences are usually accounted for during the rostering phase. The following is an example of a rostering formulation with preference considerations:

$$Min \sum \sum c_{ij} x_{ij} \qquad (1.4)$$
$$st:$$
$$\sum x_{ij} \geq b_j \ \forall\, j \qquad (1.5)$$
$$x_{ij} \in \{0,1\}$$

Where x_{ij} is a binary variable indicating that nurse i is assigned to shift j and c_{ij} is a cost penalty associated with nurse i working shift j. While similar to the set covering model for demand satisfaction this simple preference rostering model would have far more variables. The research in this dissertation focuses on this final step of the scheduling problem – rostering. And, in the case of schedule disruptions, the rebuilding of existing rosters called rerostering.

1.3 EXISTING STAFF SCHEDULING LITERATURE

This section gives a historical overview of the evolution of the staff scheduling literature. Following the general overview we focus specifically on research handling the rerostering problem and nurse preferences. A tablature outline of the research covered in this section can be found in Appendix A.

Many studies have been performed on the staffing problem. While this research covers many varying themes, the majority focus on staffing demand satisfaction and staff rostering. The associated columns of Table A-2 in Appendix A clearly shows this trend. This demand satisfaction and rostering is normally done by tour scheduling. A tour is the

combination of two problems: that assignment of staff to a given shift in a day and the assignment of days-off. The majority of research applicable to nurse scheduling can be considered tour scheduling. Tour scheduling literature has evolved since the 1970's. First there was a striving for tractability and then an increase in problem complexity as computing power improved.

Early solutions to the staff scheduling problem were more constrained than the solutions in recent literature. For example Warner (1972), Abernathy (1972), and Trived (1976) employed staff scheduling models that tried to minimize costs by allocating staff to specific wards or stations. They did not assign shifts and days-off (tours), for individuals but simply allocated staff. Miller (1976) and Warner (1976) present some early work that assigns specific individuals to tours on smaller problem sizes.

Increasing the complexity of work tours, Segal (1972) proposed a model for scheduling operators that included placement of rest breaks in the tour. Unrelated to Segal's paper, Gentzler et. al. (1977) tried to develop what they claimed to be the first quantitative model to determine the affect of work breaks on productivity. Their model was quickly adjusted by Bechtold (1979) which started an increase in tour scheduling literature that investigated the relationships between work-rest cycles.

In the 1980's Morris and Showalter (1983) openly shunned the day-off only and shift scheduling only models in favor of tour scheduling (the combination of both) showing that a simple rounding of the LP relaxation often performs close to optimal. Bailey (1985) agreed that the shift and days-off scheduling problems were obviously related and favored the tour scheduling model. He demonstrated its use for scheduling tours with shifts having variable starting times.

To handle the complexity of the tour scheduling model Glover and McMillan (1986) developed a tour heuristic designed to solve more general problems. They reported results that came within 98-99% of optimal for problems with over one million variables. Bechtold (1988) presented a multi-objective model for what he termed “implicit” optimality, or optimal for a model including only essential variables. He then demonstrated a heuristic that solves the problem within 3%-8% of optimality. Okada and Okada (1988 and 1988) and Baxter and Mosby (1988) developed computer routines focusing on speeding up manual scheduling procedures without guaranteeing optimal results.

Focusing even more on computing, Ozkarahan and Bailey (1988) developed an IP driven tour scheduling module designed to operate within a scheduling software system. Ozkarahan (1989) then extended this research to present a model for a complete scheduling software system. This is one of the earliest research efforts to fully integrate state-of-the-art tour scheduling techniques into software systems.

While others were pushing for and developing tour models, Bechtold et. al. (1984 and 1988) continued research in rest break scheduling. These papers focused on maximizing worker output by controlling work rate decaying over a work period by scheduling multiple break periods. The first model addresses limitations of earlier work-rest models and used a linear work performance decay assumption. The second paper extended this research to assume exponential work-rate decay.

In the 1990’s there is more research into better scheduling formulations, heuristics, schedule constrained algorithms and evolutionary algorithms (EAs). In the early 1990’s several heuristics are presented that use boutique algorithms or LP

relaxations (Loucks and Jacobs 1991, Easton and Rossin 1991, Franz and Miller 1993, Thompson 1993, Bechtold and Brusco 1994, Brusco and Johns 1996, Brusco and Jacobs 1998). Working sets were commonly used to simplify the solution space for the tour scheduling problem (Easton and Rossin 1991, Bechtold and Brusco 1994, Bechtold and Brusco 1995). Working sets limit the number of schedules to be considered before attempting to solve the problem. Another means to simplify solving the tour scheduling problem is to constrain the problem to specific schedule types. Hung (1991, 1993, 1994, 1994, 1994), Hung and Emmons (1993) and Burns and Narasimhan (1999) present a set of algorithms that develop schedules based on three-day work weeks, four-day work weeks, and flexible or rotating combinations of both.

In the mid 1990's the development of heuristics for tour scheduling problems changed. Brusco and Jacobs (1995) and Thompson (1996) present two simulated annealing heuristics heralding the arrival of evolutionary algorithms in the tour scheduling literature. In the late 90's Dowsland (1998) and Easton and Mansour (1999) respectively presented a tabu-search and genetic algorithm. The majority of tour scheduling heuristics now will use EAs.

Focusing on faster and better optimal solutions, Millar and Kiragu (1998) presented a network formulation for the tour problem that combined work days into consecutive sets called stints. They contended their formulation allowed for easier inclusion of various constraints than in traditional formulations. Brusco and Jacobs (1998) took a more direct approach and improved the set covering formulations by developing a method to remove redundant columns. This method reduced the column number in their experiments by as much as 56%. Brusco (1998) further improved the

general set covering formulation by showing that his use of the dual all integer cutting plane could result in significant run time improvement.

Bechtold (1991) and Bechtold and Thompson (1993) extend earlier research in work-rest cycles to include models for the placement of fixed duration rest periods for individuals and flexible duration rest periods for groups. Bechtold's research on rest periods is finally merged with the general tour scheduling literature in an integer program (Bechtold and Jacobs 1990) and later extended by Thompson (1995).

Okada (1992) extended his logic programming research by publishing a generalized tour scheduling software program. Ozkarahan (1991) also presented a plan for a decision support system to aid schedulers in developing nurse rosters. Other scheduling systems were proposed by Lauer et. al. (1994), Randhawa and Sitompul (1993), and Chen and Yeung (1992).

With increasing computing power, the tour scheduling literature tackles an increase in problem complexity. Bard and Purnomo (2005) present an IP based heuristic model for staffing that is designed to be solved every 24hrs to account for demand variations from shift to shift. Wright et. al. (2006) adapts the staff scheduling and planning model of Abernathy et. al. (1973) to handle a larger set of considerations including overtime, multiple nurse types, varying shift lengths, nurse-patient ratios for each nurse type, patient arrival rates, patient service rates, and time dependent violations of staffing ratios. Bard (2004, 2004) and Qi and Bard (2006) published a set of literature on scheduling mail handlers at the U.S. Postal Service (USPS). This series of research include two complex IP models one of which includes two worker levels, rest breaks, partial shift assignments, various staff costs, full time workers, and part time workers.

With increased reliance on computer technology new techniques emerge in tour literature such as artificial intelligence (AI). Winstanley (2004) developed an agent-based rostering framework that breaks down the scheduling problem into subcomponents. Allowing for better decomposition and focus on solving pieces of the problem. Fung et. al. (2005) presents a complete guided search solution that combines a tree based search solver and a simplex solver. Beddoe and Petrovic (2006) use a case based reasoning (CBR) approach that stores solutions to past rostering problems and utilizes that knowledge to solve similar current problems.

The use of EAs becomes popular in tour literature, including the research of Ferland et. al. (2001), Dias et. al. (2003), Dowsland and Thompson (2000), Aickelin and Dowsland (2000), Kawanaka et. al. (2003), and Jan et. al. (2000). As the use of EAs was developed new methodologies are designed to better solve the tour problem were presented. Aickelin and Dowsland (2004) demonstrate a fast and flexible indirect genetic algorithm (GA) that encodes the problem formulation and then performs the GA's evolution on the encoded version of the population. Aickelin and White (2004) develop a method for comparing and fine tuning scheduling algorithm performance. Gutjahr and Rauner (2007) present an ant colony optimization (ACO) algorithm to try and better handle the highly constrained nature of the nurse scheduling problem.

While the state of tour scheduling literature has evolved, there are still areas that are lacking. The two areas focused on in our research are the perception of fairness when considering nurse preferences and the rerostering of nurses. Table A-2 indicates many research efforts that have tried to account for staff preference considerations.

The most common method for handling staff preferences is to apply a penalty coefficient for violations of an individual's requests (Miller et. al. 1976, Ozkarahan 1991, Franz and Miller 1993, Dowsland 1998, Dowsland and Thompson 2000, Aickelin and Dowsland 2000, Ikegami and Niwa 2003, Kawana et. al. 2003, Aickelin and White 2004, Aickelin and Dowsland 2004, Belien and Demeulemeester 2006, Gutjahr and Rauner 2007). This can be applied directly to the entire work schedule sequence or individual days. One of the earliest preference models to use penalty coefficients is Warner's (1976) paper that allowed each nurse to allocate 50 penalty points to represent aversions to certain schedule characteristics.

Another common method is to consider those schedule characteristics that are preferred by the entire nurse staff (rather than individual preferences) and to develop constraints that either completely enforce those characteristics or weigh them with penalties. These schedule characteristics may include the number of consecutive workdays, the number of consecutive days off, and the number of days worked (Azaiez and Al Sharif 2005, Bard and Purnomo 2005, Bard and Purnomo 2007, Gutjahr and Rauner 2007).

Since it is often difficult to determine and compare the weights assigned to the preferences of different nurses some authors rely on a count of the total preference violations. Accordingly, Chen and Yeung 1992, Wright et. al. (2006), and Dias et. al. (2003) tried to minimize the total number of preference violations in a final staff roster. Feeling that this total count metric does not ensure nurses are treated fairly, Bard and Purnomo (2005) introduced a variation that uses an exponential weighting based categorical preferences.

Preferences can be considered in other ways as well. Winstanley (2004) assumed that when a request for a day off was granted it became a hard constraint. Bard and Purnomo (2005) and Miller et. al. (1976) introduced ways to ensure parity from one scheduling period to the next. Okada and Okada (1988) used a logic programming method to assign preferences based on a predefined rule set.

The rerostering problem is common in the airline and railroad industry. In these sectors disruptions to planned schedules are common due to weather, construction and accidents. The research in these areas deal with replacing passenger travel itineraries and the redeployment of aircraft and crews (Thengvall et. al. 2000, Rosenberger et. al. 2003, Yu et. al. 2003, Bratu and Barnhart 2006, Huisman 2007). Schedule recovery research is also in the service sector and focuses on maintaining minimum service rates by rescheduling staff (Easton and Goodale 2005). While these problems are related to the nurse rerostering problem, they are too different to be directly applicable.

The first nurse rerostering research was published by Moz and Vaz Pato in 2003. In this paper the authors model the rerostering problem as a multi-commodity flow network. Their goal is the minimizing the total number of disruptions to the initial roster. Their research was extended to include two more multi-commodity flow models (Moz and Vaz Pato 2004) and a GA approach (Moz and Vaz Pato 2007). Hattori et. al. (2005) published the only other nurse rerostering solution we found. Their solution is based on dynamic constraint satisfaction.

1.4 EXISTING AGENT-BASED SCHEDULING LITERATURE

This section gives a brief summary of how agent-based programming methods have been used in the past scheduling research. The focus of this section is on agent-based scheduling methods and does not include other applications of agent software.

Agent scheduling methods have been used for a variety of scheduling problems including transportation scheduling (Fischer 1996), meeting scheduling (Chun, 2003)(BenHassine and Ho, 2007), staff scheduling (Winstanley, 2004), project scheduling (Knotts et. al. 2000)(Confessore et. al. 2007), and manufacturing and production scheduling (Shen 2002, 2006).

Agent-based methods represent an alternative to traditional centralized artificial intelligence methods such as Genetic Algorithms (GAs), and neural networks (Shen et. al. 2006). Unlike these traditional AI solutions, agent systems decentralize problem solving. Agent systems have several advantages over centralized AI solutions to include, the natural ability to parallelize computation, agents can be attributed to actual devices or persons to realize real-time rescheduling, and agents can integrate other solution methods to help with problem solving (Shen 2002).

Both Winstanley (2004) and BenHassine and Ho (2004) cited the intuitively distributed nature of personnel scheduling as a motivator for selecting an agent-based approach. We chose an agent-based approach when conducting our research to leverage the same natural problem distribution. Furthermore using an agent system allows us to develop a real-time system that not only rosters but rerosters nurses.

1.5 KEY PAPERS

There are three papers that are key to our research. The first was published by Jonathan Bard and Hadi Purnomo in 2005 and is titled "Preference Scheduling for Nurses Using Column Generation." In this paper Bard presents an algorithm that mixes self-scheduling with traditional IP methodologies. The goal is to minimize float nurses and costs while producing good schedules that account for nurse preferences.

Bard's algorithm starts with a desired schedule based on nurse sign-ups. He requires that nurses sign-up for the shifts they prefer and then uses the resulting roster as a staring point. The key to his algorithm is getting from that starting point to a feasible roster. The algorithm is a six step process that utilizes column generation as a means to minimize the cost of a final schedule. Bard's cost coefficient is a function of both requests and preference violations.

Bard's paper is important to our research because he focuses on the perception of fairness or how the nurse's feel about the schedule. While Bard presents a complex solution methodology, we will focus our discussion on how he implements a max violation constraint and an exponential cost coefficient.

The max violation constraint is designed to prevent nurses from getting routinely bad schedules. Nurses that continually have high numbers of their preferences violated, compared to other nurses, will feel like they are being treated unfairly. To minimize this effect the max violation constraint seeks to balance good and bad schedules from one scheduling period to the next.

The max violation constraint essentially limits the number of preference violations that can be included in a nurse's schedule based on the number of violations in the previous schedule. To do this the following maximum_violations_algorithm is applied:

Step 1: Compute the average and standard deviation of the preference violations in the previous scheduling period. If this is the first period than set $V_i^{Max} = \infty$.

$$\bar{v} = \frac{1}{|N|}\sum_{i=1}^{|N|} v_i^{old} \tag{1.6}$$

$$\bar{\sigma} = \sqrt{\frac{1}{|N|-1}\sum_{i=1}^{|N|} (v_i^{old} - \bar{v})^2} \tag{1.7}$$

In equations 1.6 and 1.7, |N| is the total number of nurses, and v_i^{old} is the number of violations for nurse i.

Step 2: For all nurses $i \in N$ if v_i^{old} is more than one standard deviation over the average than V_i^{max} is altered by equation 1.8.

$$V_i^{Max} \le \lfloor \max(1, \bar{v} - \bar{\sigma}) \rfloor \tag{1.8}$$

This constraint drops the maximum number of preference violations for an individual nurse who had a bad schedule with many violations. This ensures that a bad schedule will be followed by a good one.

Additionally, Bard introduces an exponential penalty coefficient. This coefficient penalizes violations of an individual nurse's preferences based on the significance

category of the preferences being violated. This helps ensure a balance of preference violations as well as avoiding the complexities of directly weighting individual preferences. Bard thought that dissatisfaction increased non-linearly depending on the severity category.

$$C_{ij}(v) = 2^{v-1} \tag{1.9}$$

Equation 1.9 is Bard's cost coefficient for penalizing preference violations. In this equation v is the category a violated preference is in. Categories included one (simple), two (serious), three (severe), and four (extreme). Various types of preference violations were assigned to these categories to simplify the cost weighting of those violations.

Like most preference scheduling papers, Bard assumes that a preference for one day off is independent from another day off. That is, the penalty for not having day 1 off is C_1 and the penalty for having day 2 off is C_2. Often times a nurse will request two consecutive days off that are not independent. Thus the penalty for having to work day 1, day 2 or both is C_{12}. To solve this problem we will use an idea from the next key paper.

In 1998 Millar and Kiragu published a paper entitled "Cyclic and Non-Cyclic Scheduling of 12h Shift Nurses by Network Programming." This paper approaches the problem of shift scheduling in a unique way. Rather than focusing on individual days Millar uses network programming to account for sequences of consecutive days on or off. These sequences are called stints.

With the nurses working 12 hour shifts they can only work either the day or night shift each day. The 12 hour shift helps simplify the stint concept by allowing each day to have only one of three values – day, night, and off. Millar assumes that nurses cannot work more than 4 days in a row or 3 nights in a row and cannot work back-to-back shifts.

Nurses also may not have more than 4 consecutive days off. Thus a stint would be defined as a sequence of consecutive work days or off days. The stints available under these scheduling rules are laid out in Figure 1-1.

D	N	DNN	DDDN
DD	NN	DNNN	
DDD	NNN	DDN	
DDDD	DN	DDNN	

There are 13 possible work stints in Millar's model.

O	OO	OOO	OOOO

With the 4 off stints there are total of 17 stints in Millar's model.

Figure 1-1: Millar proposes 17 possible sequences of days off and of days on. These sequences called stints are then used to develop a network programming model for the nurse scheduling problem.

Using the stints as nodes in a network Millar formulates two mathematical models: one for non-cyclical schedules and one for cyclical schedules. In the interest of brevity we will discuss the non-cyclic model only. This model has two terms, one for the cost of the schedule and one for the day/night balance of the schedule.

$$Min\sum_{K}\sum_{A} C_{ka} X_{ka} + \sum_{K} P_k Z_k \qquad (1.10)$$

Equation 1.10 is Millar's objective function. Here each nurse, k, is assigned to an arc, a, in the network. The stints they work are defined by the arc's starting node. The cost associated with employing each nurse to work a given stint is C_{ka}. The Z term is the

imbalance of more night than day shifts. This formulation does not account for preferences or fairness beyond the balancing of night and day shifts.

While there are a number of constraints in addition to the objective function, the important part of Millar's paper that we will borrow for our research is the stint concept. Nurses often request multiple days off in a row. This can be a simple desire for a prolonged break or a calculated effort to plan a multi-day event. In this latter case the stint concept becomes valuable. Using stints we can apply preference considerations to a block of days rather than just one day or shift.

The third key paper is Moz and Vaz Pato's "An Integer Multicommodity Flow Model Applied to the Rerostering of Nurse Schedules." Unlike the first two key papers, this focuses on the rerostering problem. To solve this problem Moz sets a multicommodity flow model using an IP formulation.

Moz defines optimal as the solution that requires the least number of changes to the current roster. To solve for this optima, Moz's formulation has an objective function with one term, cost. Equation 1.11 is a simplification of Moz's formulation that is more readable.

$$\sum_{d \in D} \sum_{a \in A} \sum_{v \in V} C_{a_d}^{v} X_{a_d}^{v} \tag{1.11}$$

This formulation is similar to Millar's in that the assignment of nurses to shifts is done via the arc of a network. In equation 1.11, $X_{a_d}^{v}$ is a binary variable indicating whether nurse v is assigned arc a between a shift on day d and a shift on day d+1. $C_{a_d}^{v}$ is the cost associated with an arc assignment. Critical to the behavior of this objective function is how Moz defines the cost coefficient.

$$C_{a_d}^{v} = \begin{cases} 0 & if\ nurse\, v\ is\ already\ assigned\ to\ the\ shift\ arc\ a\ ends\ at \\ M & if\ nurse\, v\ cannot\ be\ assigned\ to\ arc\ a \\ 1 & if\ nurse\, v\ is\ not\ currently\ assigned\ to\ the\ shift\ that\ the\ arc\ ends\ at \end{cases} \qquad (1.12)$$

This cost coefficient counts the changes in a schedule. The goal is to have as few changes in the schedule as possible. While Moz's solution works in some cases we do not agree with her definition of optimal. When considering preferences Moz's model may regard better solutions to be too disruptive. It is possible for a situation to arise where the best solution results in more preference satisfactions but a greater number of schedule disruptions. In this case Moz's optimal definition is incorrect. More properly stated, the optimal solution is the one that results in the least negative disruption to the nurses.

1.6 PROPOSED SOLUTION

Winstanley published the only agent based nurse scheduling paper we found (Winstanley, 2004). Winstanley recognized that both an agent framework and the nurse scheduling problem is naturally distributed. Using distributed intelligent agents in a cooperative framework accurately models the reality that each nurse and the organization has their own set of goals. In a related problem, BenHassine and Ho cite the intuitively distributed characteristics of personnel scheduling as a motivating factor for their agent-based meeting scheduling system (BenHassine and Ho, 2007).

Like the aforementioned agent based solutions, we propose a series of agent based nurse rostering algorithms to model the intuitive distribution of the problem. Any algorithm in this series is called a Competitive Nurse Rostering Algorithm (CNRA). The set of CNRAs include Competitve Nurse Rostering (CNR), Competitive Nurse Rostering

with Iterated Local Search (CNR-ILS) and Competitive Nurse Rostering and Rerostering (CNRR).

Traditionally the organizational cost minimization problem is solved using a tour scheduling IP to satisfy nurse demand predictions. Often authors will include preferences in this model as either "soft" constraints or as additional costs in a single cost coefficient. This results in a complex tour scheduling model that is focused on costs and preferences. This centralized dual focus lacks flexibility and inadequately handled preferences.

When using a CNRA a simple tour scheduling cost model is solved to develop a solution that is within an organization's staffing cost tolerance. This minimal cost roster is used as an input to a CNRA. The CNRAs uses this input in an agent-based simulation to search for alternate optimal cost solutions that better satisfy nurse preferences. As agent-based solutions CNRAs afford the nurse rostering problem the advantages discussed in Section 1.4 and improves the handling of nurse preferences.

The purpose of the CNRA method is to develop better mathematical models of how nurses perceive preferences. This will allow for the production of rosters that are perceived as more fair by the nursing staff. To do this CNRA uses only one mathematical formulation, the preference utility function of the nurses. This preference function is programmed into agents representing each individual nurse in a competitive simulation. The manning constraints are programmed into the decision logic of the agent simulation and will enforce feasibility by approving or disapproving any changes to the set of shift assignments.

The CNRAs detailed in this dissertation consider three factors for nurse preferences: informal request-offs (ROs), preferred sequence length of days off, and

preference for specific days-of-the week (DOW) off. ROs are days the nurses request off but do not want to take vacation days for. The granting of ROs are not guaranteed. Preferred sequence lengths allow nurses to indicate whether they like schedules that tend to have two, three, or four days off in a row.

The three CNRA algorithms presented in this dissertation are related. The first, CNR, uses an auction to trade work shifts between nurses. The second, CNR-ILS, extends CNR to include an ILS that explores dual feasible solutions by using staffing slack to move work shifts within each nurse's schedule. The third, CNRR, extends CNR-ILS to handle the rerostering problem. All three algorithms focus on nurse preferences.

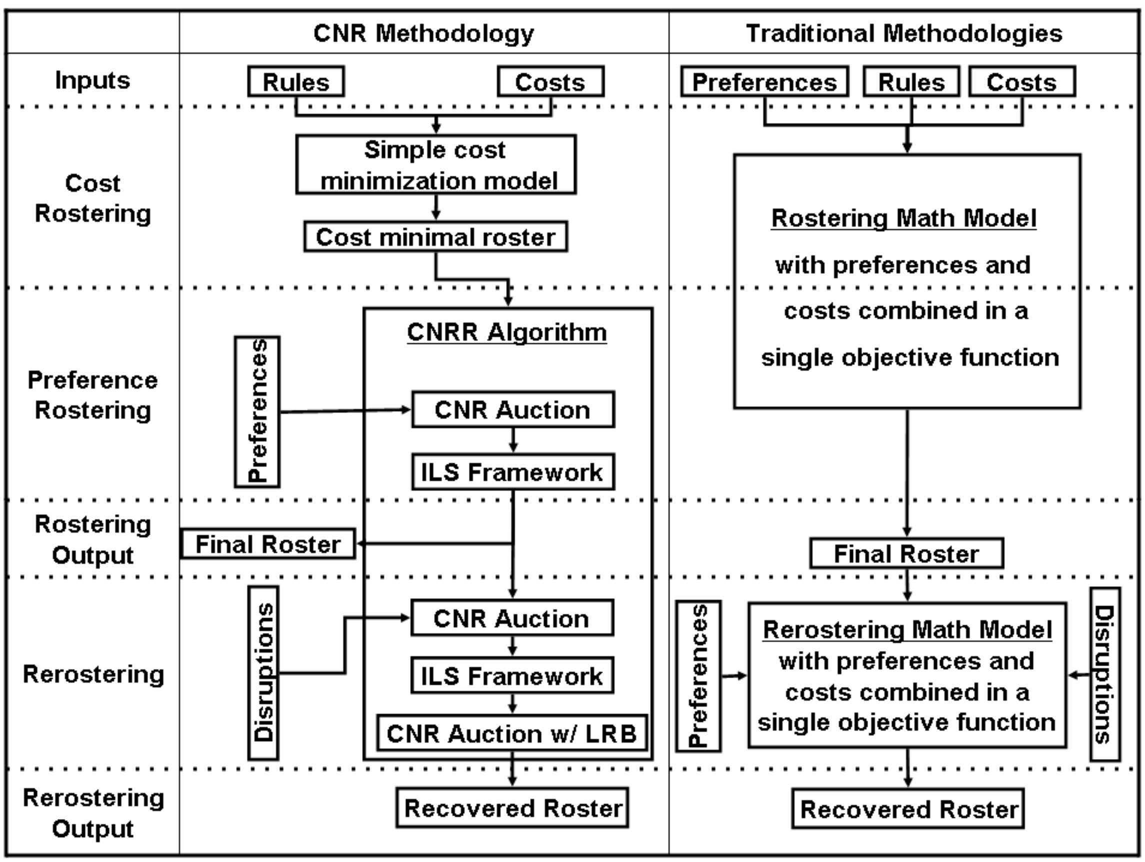

Figure 1-2: The CNRA agent model simplifies the nurse tour problem by decomposition. The CNRA model uses a simple tour assignment model to minimize the cost problem.

CNRAs take advantage of the existence of multiple rostering solutions that are cost optimal to develop a roster using an agent simulation. CNRAs avoid using the state-of-the art tour preference models that require highly constrained mathematical formulations. In this figure the CNRA represented is the CNRR algorithm which is the final product of this dissertation. The CNRR algorithm avoids the use of two complex mathematical models by implementing one simple mathematical model and one agent simulation.

The flow of the CNRA paradigm is depicted in Figure 1-2. In this figure the algorithm depicted is the CNRR algorithm, the last and most encompassing of the CNRAs. Both the CNR and CNR-ILS algorithms would not include the rerostering section in Figure 1-2. CNR would also not include the ILS Framework in the Preference Rostering section.

1.7 ORIGINAL CONTRIBUTION

The original contribution of the research in this dissertation is presented by algorithm in Table 1-1. In this table each contribution is described and which algorithm is involved in that contribution is highlighted.

Table 1-1: This table depicts which CNRA algorithm is responsible for which original contribution. Since CNR-ILS is an extension of CNR and CNRR is an extension of CNR-ILS the contributions of earlier algorithms are still present in the later algorithms.

Contribution	**CNR (Chapter 2)**	**CNR-ILS (Chapter 3)**	**CNRR (Chapter 4)**

Contribution	CNR (Chapter 2)	CNR-ILS (Chapter 3)	CNRR (Chapter 4)
Unique Rostering Methodology: CNRAs implement an agent-based model where rosters are developed through competition. There are no attempts in the literature to model the nurse rostering problem in a competitive agent based fashion.	X		
Isolate Nurse Preference Models: CNRAs use a separate utility function to model the preferences of each nurse being rostered.	X		
Separates the Organizational Cost Model and the Nurse Preference Model: CNRAs focus on maintaining cost feasibility while focusing on preference improvement	X		
Deliberately Searches Dual Optimal Solutions: Some CNRAs perform a local search of dual feasible solutions to find alternate optimal cost solutions that better satisfy nurse preferences.		X	
Unique Distributed ILS: The ILS framework in some CNRAs is designed to be an iteration of smaller distributed ILSs.		X	
Unique Rerostering Methodology: No other system has been published that can both roster and reroster nurses. Furthermore, no agent-based nurse rerostering solution has been published.			X
Considers Two Rerostering Optimality Definitions: When rerostering, solutions are restricted to those that minimize the number of shift assignments that have been changed. These solutions are found while focusing on minimizing the negative impact to nurse preferences.			X

CHAPTER 2 COMPETITIVE NURSE ROSTERING

CNR is designed to model a phenomenon often seen in shift work – shift trading. When a nurse wants a day off that is not in their schedule she will often solicit other nurses to trade shifts. CNR automates this process in a computer system designed to improve simple rosters with respect to nurse preferences. CNR starts with an initial roster that is feasible and is treated as if it is minimally staffed. The implementation of CNR described in this paper is designed for the Mike O'Callaghan Federal Hospital's Air Force Medical Surgical Unit (MOFH-AFMSU). MOFH-AFMSU rosters its nurses to 12-hr day and night shifts. All the nurses in the ward are Registered Nurses (RNs) trained in-house to one of three levels: level one nurses include new nurses who still require supervision, level two includes nurses that are no longer new to the unit but have not yet been cleared for charge nurse duty, level three nurses are cleared for charge nurse duty. Charge nurses are responsible for all nursing activity in the ward for the shift they are assigned and must be present at all times.

The algorithm relies on two types of software agents: an Auction Control Agent (ACA) and Nurse Broker Agents (BAs). The ACA is responsible for soliciting sales, soliciting bids, ensuring the feasibility of staffing numbers, and determining algorithm termination. The BAs are responsible for modeling the preferences of the nurses they represent, deciding which shifts to sell, deciding which sales to bid on, and ensuring their individual schedules are feasible.

All work shifts are traded as stints, a concept introduced by Millar and Kiragu in 1998. They define a stint as a series of consecutive work or off day shifts. The stints available in this implementation of CNR are detailed in Table 2-1.

Table 2-1: This table lists Millar and Kiragu's (1998) stints as they are adapted to the manning rules at Mike O'Callaghan Federal Hospital. D indicates a day shift assignment, N a night shift assignment and O indicates a day off. Thus DDN is a stint of three consecutive days where a nurse works two day shifts followed by a night shift.

OFF DUTY STINTS	**ON DUTY STINTS**		
O	D	N	DN
OO	DD	NN	DDN
OOO	DDD	NNN	
OOOO			

2.1 THE AUCTION CONTROL AGENT

The ACA is responsible for controlling the flow of the CNR algorithm. During the initialization of the CNR algorithm the ACA initializes control variables and receives information on the required demand levels for each shift. Once the ACA is initialized it reads the nurse roster from a data file. Using this initial roster, the ACA creates the data elements that store the current staffing levels in the initial roster and creates the BAs that are required to represent all the nurses in the roster. A sequence list in the ACA maintains the BAs in a specific order determined by the order of the nurses in the initial roster. Finally the ACA passes initializing information to the BAs.

The ACA is composed of two major datasets: the demand picture and the current auction offer. The demand picture is stored in four arrays, two for each the day and night shifts. The day shift arrays are depicted in Figure 2-1. The four arrays consist of four sets

of variables: the minimal day shift staffing (D_{rj}), the day shift's current staffing (Ad_{rj}), the minimal night shift staffing (N_{rj}), and the night shift's current staffing (An_{rj}). In these variables r is the training level from the set of the three in-house nurse training levels and j is the day of the scheduling period from the set of days in the scheduling period (J). Thus D_{rj} is the minimum number of nurses trained to level r that are required to work day j. Nurses of lower training levels can be substituted by nurses of higher training levels.

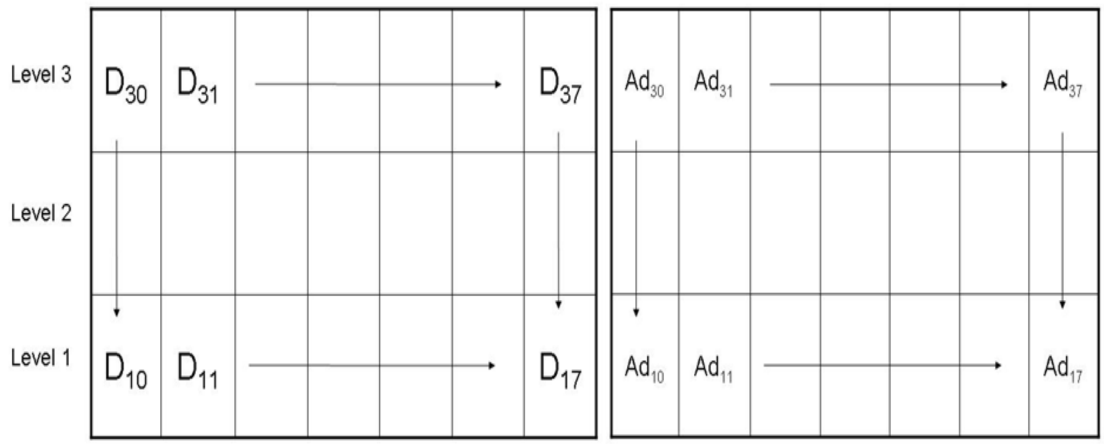

Figure 2-1: This represents the day shift demand picture arrays for a one week period. Each shift traded in the CNR algorithm must satisfy the minimal demand requirements for every day of the scheduling period (j) and every nurse level (r). In this figure Drj is the minimal number of nurses trained to at least level r that are required to work the day shift on day j. Adrj is the current total number of nurses at level r assigned to the day shift on day j of the scheduling period.

The auction offer is a collection of stints; one stint is the set of shifts being auctioned off, called the sale item, the other stints are stored in a list called the currency. All the stints in the auction offer are on-duty stints. By using this auction offer format nurse broker agents are trading workdays. The auction offer is depicted in Figure 2-2.

	Start Day	End Day	Shift 1	Shift 2	Shift 3	Shift 4
Sale Item	5	7	D	D	D	/
Currency	12	14	D	D	D	/
	1	3	D	D	D	/
	/	/	/	/	/	/
	/	/	/	/	/	/

Figure 2-2: The auction offer is used to pass the stint that is for sale (the sale item) to the bidding BAs. The auction offer contains a sorted list of stints (currency list) that the selling BA is willing to take in trade for the sale item. The currency list is sorted from best to worst so that trading the stint at the top of the list for the sale item will improve the seller's schedule the most.

The auction offer's currency list is used to determine the highest bid in the auction. When developing the auction offer the selling BA will place every stint that they are willing to accept in trade for the sale item into the currency list. The currency list is then sorted so that the trade that will result in the greatest improvement to the selling nurse's schedule is listed first. The bid highest up on the list is awarded with a sale.

Aside from initialization, the ACA has four major functions: soliciting bids, soliciting sales, closing sales, and closing the auction. The ACA runs a sequential auction. In this auction, sales are solicited from one BA at a time. At its completion a new sale is solicited from the next BA. To control the sequence of the auction the ACA tracks the sequence of all the BAs in a list. Sequencing instills the CNR algorithm with a hierarchy present in the workplace. This hierarchy allows the model to give nurses with

more seniority the ability to attempt sales first when most nurses will still have shifts to trade.

When soliciting a sale the ACA passes an empty auction offer to the solicited BA. After the BA returns the auction offer to the ACA there are two tasks that the ACA must perform: first, it must verify there is a valid sale item; second, if there is a sale item, it must check the staffing demand associated with that sale item. To verify that a valid auction offer was returned the ACA will receive a control flag from the BA indicating if the BA has a stint to sell. If the flag returns false then the ACA will solicit a sale from the next nurse in the sequence.

When the ACA receives a valid auction offer it prescreens the bidders to help ensure that nursing demand levels are maintained. To do this the ACA checks the training level of the selling nurse and the current staffing. During this check the ACA looks for the following conditions in the demand picture:

1) $Ad_{rj} > D_{rj}$ or $An_{rj} > N_{rj}$ where r is the selling nurse's level and for all j in the set of days in the sale item.
2) $Ad_{rj} = D_{rj}$ or $An_{rj} = N_{rj}$ where r is the selling nurse's level and for all j in the set of days in the sale item.

The first condition indicates that there are more nurses of the selling nurses training level than are required. This does not mean that there are more nurses working than are needed. For example, a shift that requires five nurses and only one of those nurses is required to be a level three nurse may have five nurses working and two are

level three nurses. In this case there is a surplus of one level three nurse. When there is a surplus of nurses at a specific training level and one of those nurses is attempting to sell shifts at auction, the ACA will open the auction up to nurses of all levels.

The second condition indicates that the shifts being sold have the minimum number of nurses assigned that are trained to at least the same level as the selling nurse. Under this condition the ACA opens the auction up to only those bidding nurses that are trained to at least the same level as the selling nurse. This ensures that the nurse that is no longer working those shifts is replaced by a nurse that is trained to at least the same level.

After obtaining an auction offer the ACA opens the auction up to bids from the nurses that are feasible after checking demand conditions. These bidding nurses are approached by the ACA in the order they appear in the sequence list. In this list, the selling nurse is always first. The first potential bidding nurse is second. When soliciting a bid, the ACA will supply the bidding nurse with the auction offer and the index of the current highest bid in the currency list. The BA will indicate if they are bidding, if they are bidding they will the include index of their bid in the currency list.

Once a bid is received the ACA must perform another check of the demand picture. This check is exactly the same as the check performed after soliciting a sale except that it is performed on the bid, not on the sale item. If the bid should fail the demand check the ACA will solicit the same nurse again for a new bid.

After the first pass through the sequence of bidding nurses, the ACA will check to see if there is a highest bid. If there is a highest bid, the ACA will solicit new bids from all the bidding nurses except the nurse that placed the current highest bid. These new bids

must be higher than the current highest bid. This process will be repeated until a pass through the sequence of bidding nurses results in no change to the highest bid.

Once the bidding has ended the ACA will inform the bidding and selling BAs and the BAs will then update their schedules. The ACA will also adjust the demand picture as necessary. After the sale is closed the ACA will move the selling nurse to the end of the nurse sequence and the next nurse will become the new selling nurse. The ACA's auction control algorithm is presented in pseudo code in Figure 2-3.

```
initialize auction;
LOOP through BAs to solicit sales for the maximum iterations or no more potential sale items exist

    solicit sale from current BA;
    IF BA returned a valid sale item THEN
        check staffing demand conditions;
        IF Sale item is minimally staffed at the selling nurse's training level THEN
            set min bid level to the selling nurse's level;
        ELSE set the min bid level to lowest training level;
        END IF

        LOOP through every BA except the selling BA until every bidder fails to create a new high bid
            IF the current BA is not high bidder and is trained to at least the min bid level THEN
                solicit bid from current BA;
                IF BA returns a bid THEN
                    check staffing demand conditions for the bid;
                    IF Bid is minimally staffed at the bidding nurse's training level THEN
                        IF bidders level ≤ current seller's level THEN
                            update highest bid;
                        END IF
                    ELSE update highest bid;
                    END IF
                END IF
            END IF
        advance to next bidder;
        END LOOP

    END IF
    advance to next seller;
END LOOP
close auction;
```

Figure 2-3: This figure depicts the ACA auction control algorithm in pseudo code

2.2 THE NURSE BROKER AGENTS

The broker agents (BAs) contain the decision logic that is used to model each nurse. They are responsible for generating auction offers, generating bids, and ensuring the feasibility of their individual schedules. The BAs are created by the ACA and receive their initialization information from the ACA and data files.

The BA contains two major data structures: their preference information and a pair of lists called the give and take lists. Nurse preference information is stored as a single data structure. It contains preferences for requests off (ROs), preferences for having specific days of the week (DOW) off, and preferences for the length of off-duty stints. The preference data structure is represented in Figure 2-4. These preferences are obtained from surveying the nurses prior to developing a roster using the survey in Appendix A. While preferences for ROs change from roster to roster, preferences for DOW and the length of off-duty stints tend to change less frequently. As a result we considered surveying and inputting preferences for DOW and off-duty stints less often or only when a nurse requested it. In practice we surveyed and inputted all the preference values before developing each roster. This was done because the point values assigned for each preference are evaluated relative to each other and while a nurse may desire three days off in a row in every roster, the relative rating of these preferences may change.

Start Day	End Day	Weight
1	2	R_1
7	7	R_2
20	20	R_3
/	/	R_4

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
P_{sun}	P_{mon}	P_{tue}	P_{wed}	P_{thu}	P_{fri}	P_{sat}

4 Days Off	3 Days Off	2 Days Off
Q_4	Q_3	Q_2

Figure 2-4: The nurse broker agent (BA) stores a nurse's preference information in a set of arrays. CNR includes nurse preferences for requested stints off (R_k), specific days of the week off (P_d) and for preferred number of days off in a row (Q_e). These preferences are used in each BA to represent each nurse's utility function. In this figure there are three ROs, the first is from day 1-2, the second is day 7, and the third is day 20. The fourth RO has no start or end day, this indicates that no fourth request was made.

Using the preference weights from Figure 2-4, the BA represents a nurse's schedule utility function as follows:

$$U(S) = \sum_{k \in K} R_k X_k + w \sum_{d \in D} P_d N_d + \sum_{e \in L} A_e Q_e S_e \qquad (2.1)$$

In Equation 2.1, K is the set of request offs {1, 2, 3, 4}, D is the set of days of the week, L is the set of off duty stint lengths {2, 3, 4}. R_k is the weight of the nurse's

preference to have their k^{th} RO off. X_k is a binary variable indicating whether the nurse has their k^{th} RO off. P_d is the weight of the nurse's preference to have the d^{th} day of the week off and N_d is the number of days of the week d that the nurse has off. w is an adjustment factor for each day of week. This adjustment factor is discussed later. Q_e is the weight of the nurse's preference to have a schedule that focuses on having e days off in a row. S_e is the number of times the nurse has e days off in a row. The preference weights for off stints of length e are adjusted by an adjustment factor $A_{e.}$

The adjustment factor A_e, is used to ensure that shorter stints are not favored. For example a 28-day schedule can have seven two-day off stints or only three four-day off stints. Thus if a nurse indicates that they prefer two-day stints with a weighting of 10 points and four-day stints with 20 points having seven two-day stints is worth 70 points and three four-day stints is worth only 60 points. Our adjustment factor resolves this problem. The adjustment factor is defined as the reciprocal of the maximum number of off duty stints a scheduling period can have of a specific length. For a 28-day period there can be seven two-day stints, four three-day stints and three four-day stints. The adjustment factors for these stints are 1/7, 1/4, and 1/3 respectively. With the adjustment factor, the third term represents the degree to which the current schedule focuses on having stints of length e off. The days-of-the-week (DOW) term also includes an adjustment factor (w) to represent the degree to which the schedule focuses on having a given DOW off. w is set to 1/4 for a 28 day scheduling period.

The adjustment factors also play an important role in helping to ensure that ROs are granted greater importance than other schedule characteristics. At MOFH-AFMSU the nurses all expect to have their ROs granted before any other preferences are

considered. The adjustment factors reduce the potential for a single stint or DOW interfering with the algorithms drive to obtain ROs.

In addition to the preference inputs in the utility function (Equation 2.1) the BA adds on two penalties: one for single days off and one for unbalanced schedules. An unbalanced schedule is one that has a large majority of the work shifts in either the first or second half of the scheduling period. These penalties are preset within the algorithm and not solicited from the nurses as preferences. We chose not to solicit the nurses for preferences with respect to single days off and schedule balance to simplify the preference solicitation process. Since theses schedule characteristics are considered negatives, removing them from the set of inputs required from the nurses simplified the preference surveys (discussed later) to only the preferences a nurse may want to have. While we used a present penalty term to simplify our surveys, there is no algorithmic reason these penalties cannot be surveyed from the nurses. The penalty terms are added to the schedule utility function as follows:

$$-\sum_{i \in I} C_1 d_j \tag{2.2}$$

$$-\sum_{i \in I} C_2 b_j \tag{2.3}$$

Equation 2.2 is the penalty term that accounts for on-off-on work patterns. The d_j variable indicates a single day off at day j in the schedule period. The cost coefficient (C_1) is set prior to runtime. Equation 2.3 is the balance penalty term that ensures schedules are not too unbalanced or heavily skewed with workdays in the first or second half of the scheduling period. The b_j variable is a binary variable indicating that there are

five work days in a six day stretch beginning at day j in the scheduling period. The cost coefficient (C_2) is set prior to runtime. This balancing penalty helps discourage the concentration of work shifts and therefore helps spread or balance a nurse's workload over the entire scheduling period.

The give and take lists represent the potential bids and sales a BA can be involved in. Each are derived by the BA from its current schedule. The give list represents every work stint that the BA has available to trade. The take list represents every work stint that can fill the BA's current days off. Figure 2-5 is an example of a seven day schedule's give and take lists.

0	1	2	3	4	5	6
D	D	O	O	D	D	N

Give List

Start	End	Shift 1	Shift 2	Shift 3	Shift 4
4	6	D	D	N	
5	6	D	N		
4	5	D	D		
0	1	D	D		
6		N			
4		D			
1		D			
0		D			
5		D			

Take List

Start	End	Shift1	Shift 2	Shift 3	Shift 4
2	3	D	D		
2	3	D	N		
2	3	N	N		
2		D			
2		N			
3		D			
3		N			

Figure 2-5: This figure represents the give and take lists derived from the seven day schedule at the top. The give list is built by taking every stint that can be permutated from the work days that the nurse is scheduled to work. The take list is derived by creating every work stint that can fill the off days of the nurse's schedule.

After producing the give and take lists each BA will filter their take list to help the agent achieve its goals. When filtering their take list, the agents will remove all the stints that conflict with any RO. A stint is considered in conflict with an RO when it contains at least one shift on a day that is part of a RO. This filtering mechanism ensures that a BA will never agree to a trade that takes on work for any day that is part of a RO. This helps improve the ability of BAs to achieve their ROs, particularly when the ROs are multiple day stints.

Using its preference data structure and the give and take lists, each BA performs two key functions: creation of auction offers and bidding on auction offers. When the ACA solicits a BA for an auction offer the BA develops both the sale item and the auction offer's currency. To select a stint to auction off, the BA selects the stint in its give list located at the index stored in an internal tracking flag. To correctly develop auction items the BA must maintain the give list and its index tracking flag.

Since the index of the give list is used to decide which stints to try and sell first, the BA maintains the list in a specified order. This order is dependent on the weights assigned to each stint in the list. The weight for each stint is produced by adding the preference weights of all the DOWs in the stint with a weighting for all partial or complete ROs that may be part of the stint. The preference weight for each RO will be added proportionally to the amount of overlap the stint has with the RO. For example, if the stint includes one of three days of a RO then 1/3 of the RO's preference weight will be added to the give stint weight.

An index tracking flag indicates which stint should be auctioned off from the give list. This tracking flag identifies the first (best) stint in the list that has not yet been

offered for sale. To indicate failures, the tracking flag is incremented by one every time the BA fails to sell a stint. When the BA's schedule changes by either a successful auction or by a successful bid, the BA must update the give and take lists. When the BA updates its give and take lists, the index tracking flag is reset to zero. The BA auction sale algorithm is depicted in Figure 2-6.

```
set sale item = copy of the first stint in the give list that has not been placed at auction since this
      BA's last successful sale;
IF sale item is valid THEN
      LOOP through take list stints
            IF current stint does not conflict with a request off THEN
                  IF trading the current stint for the sale item produces a feasible schedule THEN
                        IF trading current stint for sale item improves the selling BA's utility THEN
                              add current stint to the currency list as a plausible trade for the sale item;
                        END IF
                  END IF
            END IF
      END LOOP
      sort currency list so that the best option is first;

      wait for ACA to solicit bids;

      IF ACA succeeds in auctioning off the sale item THEN
            set index tracking flag to 0 and swap shifts;
      ELSE Increment index tracking flag;
      END IF
END IF
end sale;
```

Figure 2-6: This figure depicts the Broker Agent's sale control algorithm in pseudo code.

When the ACA solicits a BA for a bid, the BA must determine if it can add the sale item to its schedule and if there is a stint in the seller's currency list that would result in a valid bid. A valid bid requires that the bidder's resulting schedule is both feasible and that the constraint defined by Equation 4 is not violated. In this constraint $U(X_h)$ is the utility of a schedule resulting from a hypothetical trade and $U(X_c)$ is the utility of the current schedule. B is the bid threshold. This threshold controls the likelihood for trades

to take place in the CNR auction framework. The use of a positive bid threshold will allow for trades that hurt a nurse's schedule. A positive bid threshold can be used to mimic goodwill. An example of goodwill is when a nurse trades shifts with another nurse as a favor; these trades may burden one nurse and benefit the other.

$$U(X_h) + B \geq U(X_c) \tag{2.4}$$

A BA can bid on the sale item only if it is not already scheduled to work any of the days included in the sale item. Once this determination is made the BA can try and develop a feasible bid. When determining its bid, a BA compares its give list to the currency list. Through this comparison the BA can filter out the stints that cannot be offered for the sale item. The BA can offer only those stints in the currency list that exactly match a stint in its give list. No matches indicate that the BA is not working any stints that the auctioning BA is willing to take in exchange for the sale item.

After comparing the currency and give lists the bidding BA performs hypothetical trades using the sale item and the remaining stints in the give list. When performing these hypothetical trades the BA filters out trades that result in infeasible schedules and notes the schedule improvement for all feasible trades. The BA further filters the feasible bids using the threshold criteria defined in Equation 2.4. The resulting stints are the set of valid bids. From this set the BA will select the stint that results in the greatest schedule improvement that is higher in the currency list than the current highest bid. The index of the current highest bid is passed to the BA when the ACA solicits a bid. The bid algorithm is shown in Figure 2-7.

```
IF  BA has the days off required to work the sale item THEN
    set best trade = null;
    LOOP through stints in currency list that are higher than the current high bid
        IF current stint is in the BA's give list THEN
            evaluate potential for trading current stint for the sale item;
            IF trading produces a feasible schedule THEN
                IF  the trade results in a higher utility value than the current best trade THEN
                    set best trade = current stint;
                END IF
            END IF
        END IF
    END LOOP
    IF best trade meets the bid threshold THEN
        Issue a bid for the stint;
    END IF
END IF
end bid;
```

Figure 2-7: This figure depicts the Broker Agent's bid generation algorithm in pseudo code.

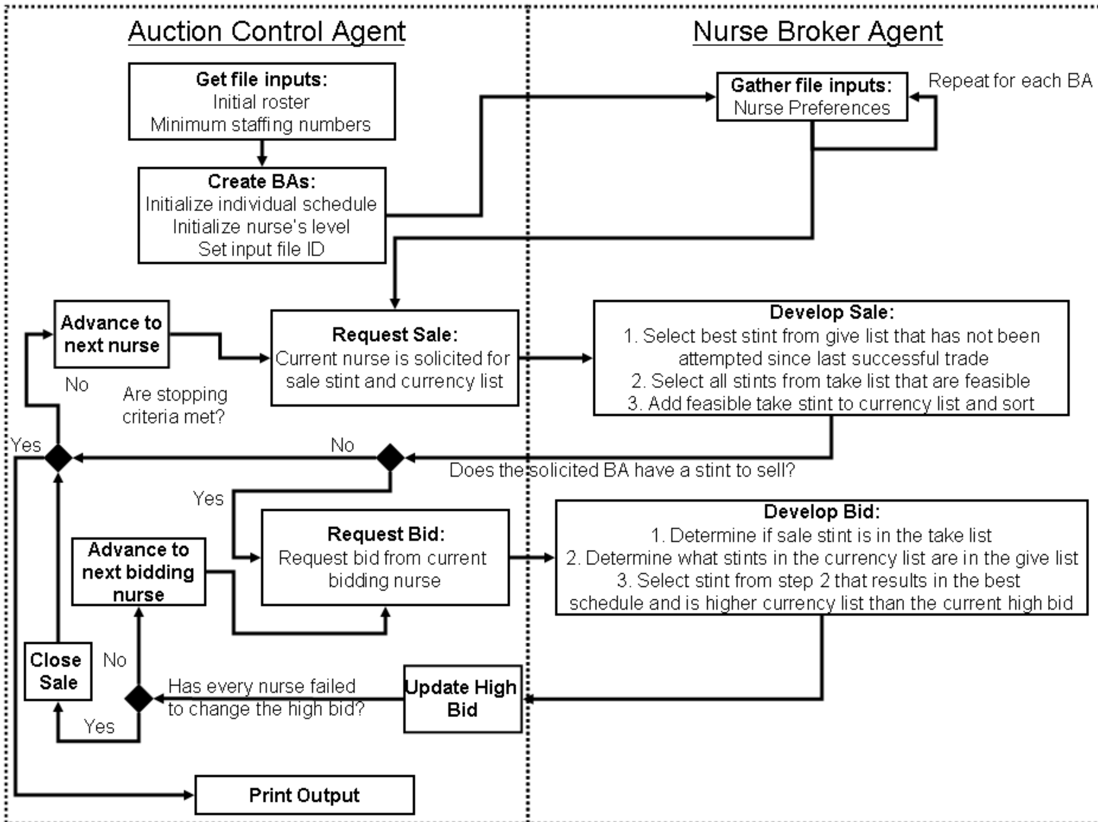

Figure 2-8: This figure depicts the whole CNR algorithm. Actions in the right dashed rectangle are performed by the BA while those in the left are performed by the ACA.

The ACA and BAs are used in the CNR algorithm to form a sequential auction framework for the trading of nurse shifts. Figure 2-8 presents a high level view of ACA-BA interactions.

2.3 CNR AUCTION CONVERGENCE

The CNR auction can be terminated by the ACA when either the schedule converges on a point when no stints can be sold or the maximum number of algorithm iterations has been reached. The algorithm has reached the maximum number of

iterations once it has solicited a sale from every BA 250 times. We selected 250 as the maximum number for two reasons: first, when the algorithm was setup to guarantee convergence, it always converged before the 250 iterations when there are 20 nurses; second, using more than 250 iterations resulted in large amounts of cycling when the algorithm was not going to converge. Cycling is the repetitive trading of a stint between two nurses. These observations are valid only for the 28-day schedules with 20 nurses that we experimented with. If there are N nurses, each iteration affords each nurse N opportunities to trade shifts. As a result the number of required iterations is not sensitive to the number of nurses. In contrast, the number of required iterations is sensitive to the number of days in the scheduling period. Increasing the length of the scheduling period increases the number of shifts each nurse can trade. This increases the potential number of trades and requires more algorithmic iterations. As a rule of thumb, the maximum number of iterations required increases linearly with the problem size in days. This linear effect assumes that the density of nurse preference considerations are constant (adding more days means each nurse may have more ROs).

Convergence is determined by two tracking flags for each BA. The first flag, G_i, corresponds to the total number of stints in a BA's give list where i is the index of the BA in the ACA's sequence list. The second flag, AG_i, is the index of the last stint that the i^{th} BA attempted to sell from its give list. Since BAs attempt to sell stints from their give lists sequentially, the ACA will only solicit a BA for a sale when $G_i > AG_i$. Whenever a BA trades shifts with another BA by either selling or bidding, the BA will create a new give list based on its new schedule. Whenever this happens the ACA will reset G_i and

AG_i for the BAs involved in the trade. When $AG_i \geq G_i$ for all the BAs in the auction, the ACA considers the auction converged and will terminate the auction.

Convergence is dependent on the bid threshold, B, used in Equation 2.4 and is not always guaranteed. BAs are designed to sell stints only when the resulting trade improves the utility of its schedule. Bidding is more flexible than selling. When bidding on stints, B controls the characteristics of the nurse utility functions. If $B \leq 0$ then the BAs will bid on a stint only when the trade results in a schedule that has a utility that is at least as high as the utility of the current schedule. This characteristic implies a utility function that is strictly increasing. In this case the algorithm will eventually converge, provided the maximum number of iterations is high enough. If $B > 0$ then a BA can place bids that will result in schedules that are worse than its current schedule. When the threshold is positive, the utility functions are not strictly increasing and convergence is not guaranteed.

2.4 MOTIVATING CASE STUDY

The experiments in this study were conducted in accordance with the staffing rules at Mike O'Callaghan Federal Hospital's Air Force Medical-Surgical Unit (MOFH-AFMSU). The MOFH-AFMSU is a 24 X 7 inpatient facility with approximately 20 registered nurses (RNs). In the MOFH-AFMSU the staffing rules are detailed in Appendix B.

Testing of the CNR algorithm was done in two phases, the first phase is algorithm tuning and the second phase is survey comparisons on real world scenarios. The first phase is designed to determine the optimal settings for the CNR algorithm. The second phase is to survey the impressions of the nurses at MOFH-AFMSU of the CNR algorithm

compared to their current by-hand scheduling method currently in use. The CNR algorithm is implemented in C++, developed using Microsoft Visual Studio 2005, and run on an 1.4GHz Intel Centrino with 512MB RAM in debug mode.

2.4.1. CNR ALGORITHM TUNING

The first phase of testing is setup as a half factorial experimental design with twenty repetitions. The design tests the effects of required shift demand (D), the bid threshold (B), the single day off penalty (C_1), and the balance penalty (C_2). The factorial setup is depicted in Table 2-2.

Table 2-2: The half factorial design for CNR includes four variables. Three of these variables are algorithm settings including the bid threshold (B), and single dayoff (C_1) and balancing (C_2) penalties. The fourth variable, the demand variable, (D) is a product of the environment in which CNR is delpoyed . D indicates how many nurses are required for each shift.

D	**B**	**C_1**	**C_2**
4	10	10	5
3	0	10	5
3	10	0	5
4	0	0	5
3	10	10	0
4	0	10	0
4	10	0	0
3	0	0	0

While the half factorial reduces the number of experiments that need to be performed it does have a problem with confounding. In this experiment the two factor

interactions are confounded. The confounded two factor interactions are depicted in Table 2-3.

Table 2-3: This table depicts the two factor interactions that are inherent in the half factorial design used in our experiments.

Interaction	**Confounding Interaction**
$D*C_1$	$B*C_2$
$D*B$	C_1*C_2
$D*C_2$	$B*C_1$

The experiments for the half factorial design were set up to schedule 20 nurses to a 28-day period with 10 nurses assigned to the day shift and 10 assigned to the night shift. The nurse preferences were generated randomly with input from the MOFH-AFMSU scheduling team. This random generation produced nurse preferences that were considered realistic by the scheduling team with respect to weekend preferences, off-stint preferences, and the number of request offs (ROs) per nurse.

The initial rosters for these experiments are standardized with each nurse working 14 days where every workday is followed by an off day. Half of the night and day shift nurses start their schedules on a work day while the other half start on an off day. An example initial roster is represented in Figure 2-9. None of the schedules in these random experiments consider vacation days or additional duty days.

```
Nurse 1  dodododododododododododododo
Nurse 2  nononononononononononononono
Nurse 3  odododododododododododododod
Nurse 4  onononononononononononononon
Nurse 5  dodododododododododododododo
Nurse 6  nononononononononononononono
   :            :            :
Nurse 19 odododododododododododododod
Nurse 20 onononononononononononononon
```

Figure 2-9: The initial schedules for the testing of the CNR algorithm were all designed to follow an on-off-on pattern. This type of initial schedule was easy to develop by hand and easy to adapt for real world tests in the MOFH-AFMSU

After running each experiment in the half factorial design on 20 sets of preferences from each nurse, the effects of each variable was determined with respect to five responses: algorithm runtime, the number of on-off-on work patterns, average percentage of ROs granted per nurse, the average utility of the nurses, and the average number of trades each nurse is involved in. Equation 2.5 represents the effects of various algorithm inputs on the algorithm runtime (RT). Equation 2.6 represents the effects on the average number of on-off-on patterns per nurse called “single offs” (SO). Equation 2.7 represents the effects on the average ratio of request offs that were met per nurse (PRO). Equation 2.8 represents the effects on the average utility of all the nurses. Equation 2.9 represents the effects on the average number of stint trades (T) each nurse is involved in.

$$\ln(RT) = 2.45 + 0.02 * C_1 + 0.15 * B - 0.01 * C_2 - 0.01 * B * C_1 \quad (2.5)$$

$$SO = 2.56 - 0.19 * C_1 + 0.045 * C_2 - 0.007 * C_2 * C_2 \quad (2.6)$$

$$PRO = 0.86 + 0.002 * C_1 + 0.005 * B \quad (2.7)$$

$$U = 42.33 - 0.65 * C_1 + 0.08 * B + 0.028 * C_1 * B \quad (2.8)$$

$$\ln(T) = 1.99 + 0.04 * C_1 + 0.28 * B - 0.02 * C_2 - 0.012 * \mathrm{B} * \mathrm{C}_1 \quad (2.9)$$

All the factors included in these equations are statistically significant with p-values less that 0.05. The logarithmic transformations seen in Equations 2.5 and 2.9 were performed to ensure that the residuals fit the normal probability plot and satisfied the linear regression assumption of constant variance.

Since we do not have control over the required shift demand levels we ignored interaction terms where D was included. After performing our DOE we set the algorithm to the following settings: B = 10, C_1 = 10, C_2 = 5. Setting C_1 = 10 was considered important. When C_1=0 the resulting schedules had too many on-off-on patterns. The MOFH-AFMSU considers more than an occasional on-off-on pattern unacceptable.

With $C_1 > 0$ it is important to properly set B. Having a $B \geq C_1$ ensures that while the algorithm considers on-off-on patterns a negative, it will not be prevented from making some trades as a result of the penalty. When the bid threshold is high enough to overcome the single day off penalty the number of trades per nurse more than tripled. This increased trade activity increased runtime but greatly increased the final utility values. When we set B=0 and C_1=10 the solutions were achieved quickly. Unfortunately the algorithm was not willing to make trades where a single day off was the result. This

limited the algorithm's ability to move effectively through the solution space and produced schedules that were not very good.

The balance penalty (C_2) was created and included at the behest of the nurses. While the large majority of nurses are not upset by having the occasional on-off-on pattern, they do resent heavy work periods such as on-on-off-on-on-on patterns citing the wear from frequent 12-hr work shifts.

2.4.2 COMPARATIVE METHODS

Using the settings determined by the half factorial design, the next phase of testing compares the CNR algorithm to the by-hand scheduling method in use at the MOFH-AFMSU. To compare these two methods, nurses in the unit were surveyed twice each month over a four month period. The first survey collected nurse preferences and is presented in Appendix A. This survey was adapted from the survey used by Warner in his mathematical model that collected nurse aversions (Warner, 1976). Our adaptation collects nurse desires.

The second survey collected each nurse's impression of the schedules developed by the CNR algorithm and by the by-hand method using a Likert scale. This scale, shown in Figure 2-10, is an 11 point scale anchored at -5, 0, and 5. These point values correspond to "Very Dissatisfied", "Indifferent," and "Very Satisfied" respectively. After schedules were developed using the CNR and the by-hand method, the nurses were presented the two schedules in a blind random order and asked to rate each one. This ensured the nurses were rating the schedules without knowledge of the schedule's source.

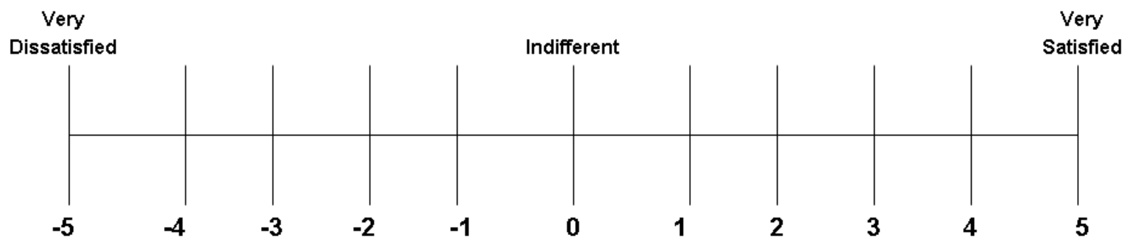

Figure 2-10: This figure depicts the Likert Scale used to survey MOFH-AFMSU nurse impressions of the schedules developed by the models under test.

2.5 RESULTS AND DISCUSSION

When properly tuned, the CNR algorithm performs well over the 20 random experimental instances. Table 4 lists the general performance characteristics of the algorithm. This table includes the average and standard deviation of each metric over all the nurses in each run. The standard deviations are reported as a measure of consistency and fairness. Fairness is measured as variability of the algorithm's performance from nurse to nurse. For example a roster that grant's an average of 90% of the request offs would be unfair if a few nurses had no requests granted while the rest had every request granted.

Table 2-4: This table presents the performance characteristics of the CNR algorithm over 20 experimental runs where the demand for level three nurses is one per shift. The data points include the average for all nurses per run and the standard deviation for all nurses per run. The bottom row is the average of the data points in its respective column. In this table all runs had 20 nurses, 10 on night shift and 10 on day shift.

Run	Avg Num Trades	Std Dev Trades	Avg Num On-Off-On Patterns	Std Dev On-Off-On Patterns	Avg Ratio of ROs Granted	Std Dev Ratio of ROs Granted	Run Time (Seconds)
1	35.7	19.47	0.2	0.52	1	0	36
2	41.2	34.7	0.5	1	0.95	0.15	35
3	53.7	39.7	0.65	0.88	0.98	11	35
4	31.6	15.9	0.35	0.59	0.93	0.24	34
5	94.1	118.04	0.3	0.57	1	0	43
6	35.2	13.63	0.1	0.31	0.98	0.11	37
7	32.3	18.42	0.3	0.57	1	0	32
8	62.1	71.09	0.3	0.66	1	0	39
9	64.7	78.11	0.45	0.69	0.98	0.11	41
10	41	21.54	0.35	0.59	0.9	0.26	36
11	47.9	20.26	0.4	0.6	1	0	41
12	30.4	12.46	0.65	0.88	0.98	0.07	34
13	34.4	15.58	0.4	0.68	1	0	35
14	53.7	31.5	0.35	0.59	1	0	40
15	39.6	13.2	0.4	0.6	0.93	0.23	39
16	47	37.97	0.2	0.55	0.98	0.11	35
17	43.8	32.09	0.35	0.59	0.98	0.11	36
18	45	20.4	0.3	0.57	0.9	0.31	38
19	46.6	46.12	0.6	0.75	0.9	0.26	37
20	34.2	26.21	0.4	0.6	0.95	0.15	30
Over All Runs	45.71	15	0.38	0.64	0.97	0.11	36

The first metric presented in Table 2-4 is the number of trades the nurses are involved in. Since this number includes both bids and sales, dividing the number in half gives the number of actual sales that the auction performed. The number of trades is a measure of the algorithm's activity. More trades generally means that the algorithm experienced more movement through the solution space. More trades are not always a positive. Run 5 in Table 2-4 is an example of more trades actually being a negative. In this run the average and standard deviation is unusually high. This is an indication of cycling. Since the algorithm allows for bids that reduce schedule utility, a situation can arise where two BAs trade a stint back and forth until the algorithm reaches its maximum

number of iterations. This cycling artificially increases the total number of trades for the BAs involved. The resulting inequity drives up the standard deviation.

The next metric in Table 2-4 is the number of on-off-on patterns. This pattern is considered a negative. The number of single days off shown in Table 4 is considered acceptable by the scheduling team at MOFH-AFMSU. Higher standard deviations indicate unequal distribution of on-off-on patterns amongst the nurses. If this distribution is too uneven, rosters may be viewed as unfair.

The next metric is the average ratio of ROs granted. The CNR algorithm failed to satisfy only one RO during the four months of testing at MOFH. In the random tests, the algorithm failed to satisfy ROs in most of the experimental runs. This difference can be attributed to the weighting of ROs in the two experiments. While the random experiments were designed to mimic the number of ROs in a scheduling period, it has a greater spread of weights than the tests at MOFH-AFMSU. At MOFH-AFMSU, nurses tend to use the highest weights they can for their ROs. In the random experimental runs some of the ROs may have small preference weights. These small weights are the cause of many unsatisfied ROs.

The random experiments were designed to that at least two-thirds of the nurses have ROs. Any nurse that does not have a RO is considered to have all their ROs granted. In all 20 random experiments at least 18 of the 20 nurses had all their ROs granted. Values in the high 0.90's usually indicate one or two ROs were unsatisfied. Values in the lower 0.90's may indicate anywhere from 1-8 ROs that were unsatisfied. These lower values usually indicate that the ROs that were unsatisfied are from nurses who only had one or two ROs.

The standard deviation of the ratio of ROs granted is reported as a fairness metric. Higher standard deviations indicate that some nurses are not having any ROs granted while others are having all of their ROs granted. This may lead nurses to perceive the rosters as unfair.

Table 2-5 presents the performance information for the CNR algorithm when the number of nurses is reduced from five per shift to four per shift. Two nurses were removed randomly from working both the night and day shifts. With fewer nurses there are fewer potential trades. This hindered the algorithm's potential to move through the solution space and reduced the performance on all metrics reported in that Table.

Table 2-5: This table presents the performance characteristics of the CNR algorithm as the average of each metric over 20 experimental runs where the demand for level three nurses is one per shift. In this table all the runs had 16 nurses, 8 on night shift and 8 on day shift.

Avg Num Trades	**Std Dev Trades**	**Avg Num On-Off-On Patterns**	**Std Dev On-Off-On Patterns**	**Avg Ratio of ROs Granted**	**Std Dev Ratio of ROs Granted**	**Run Time (Seconds)**
40.3	38.6	0.95	1.28	0.91	0.24	25.1

Table 2-6 compares the nurse utility values when there are 20 versus 16 nurses. The values are for the same 16 nurses that were in both the four per shift and five per shift random experiments. As expected, the difference in utility is statistically significant. This indicates that performance with respect to utility degrades when the number of nurses decreases.

Table 2-6: This table presents the average difference in utility values for all nurses over all runs of the CNR algorithm when there is 20 nurses and 16 nurses. The difference is calculated as the 20 nurse case minus the 16 nurse case. Only the 16 nurses used in the 16 case are considered from the 20 nurse case. This difference is significant with a p-value of 0.00.

Average Utility Difference for CNR With 20 and 16 Nurses	**P-Value for Equality of Means Test**
8.16	0.00

Testing of the CNR algorithm at Mike O'Callaghan Federal Hospital was performed over four months. The results of nurse satisfaction ratings are shown in Table 2-7. Unlike CNR, the MOFH-AFMSU's by-hand method can leverage shifts where more nurses are assigned to work than is required. Even with this advantage, the MOFH-AFMU's rostering method could not outperform CNR. This result shows that the CNR method is competitive with respect to final roster quality. In addition, the runtime of the CNR algorithm produces a schedule in less than one minute. This runtime is small enough to be practical if applied in a real word system. By contrast, the by-hand method usually takes the MOFH-AFMSU scheduling team at least six hours.

Table 2-7: This table presents how the CNR and MOFH By-Hand methods performance in nurse satisfaction surveys. While the CNR method appears to be out performed, the test on equality of means cannot distinguish between the two methods.

	CNR	**MOFH-AFMSU By-Hand**
Average Rating	1.2	1.7
Rating Standard Deviation	3.1	2.8
Percent of Responses that are Negative	33%	25%

P-Value Test Of Equality	0.4

While our test results are promising, there are concerns with the surveying process. The ratings of each schedule are framed within the context of the other schedules. Often we saw situations where the first schedule rated by a nurse was later changed after the nurse rated the second schedule. This is evidence that a schedule's rating was framed by the nurse's perception of the other schedules they were rating. Thus a nurse may be unsatisfied with a good schedule because she prefers the other schedule she is asked to rate. We attribute this to a sense of regret, the nurse knows she could have had the better schedule and therefore is compelled to dislike a schedule that, when taken in isolation, would have been satisfactory to her.

Another concern is the tendency for discrepancies between a nurse's rating and their stated preferences. Occasionally nurses would rate schedules in a fashion that seemed nonsensical when considering their stated preferences. Nurse may rate schedules that gave them everything they asked for with a negative value or rate schedules that gave them more of what they asked for lower than a schedule that gave them less of what they asked for. We attribute this to three causes. First, nurses are rating schedules approximately two weeks after they submitted their preferences and those preferences may have changed. Second, the magnitude of the two penalties (balance and on-off-on patterns) was the same for all the nurses. As a result the impact of these penalties in the utility functions may not be inline with the perceptions of each individual nurse. Third, surveying perceptions and preferences is inherently inexact and is affected by bounded rationality.

The CNR algorithm is a heuristic. CNR produces good rosters but does not guarantee that the output roster is Pareto-efficient with respect to preferences. Furthermore the performance with respect to organizational cost is dependent on the input roster; if the input roster is not cost minimal, than the roster produced by CNR may not be Pareto-efficient with respect to costs. This problem can be mitigated by altering CNR's handling of costs by including a roster cost function. This would allow the algorithm to measure the impact of any shift trade on roster costs. The auction can than limit any trades to those that either reduce roster cost or are cost neutral.

Overall the nurses at MOFH-AFMSU were pleased with the results of the CNR algorithm. While it is obvious the CNR improves the time it takes to produce a roster by-hand by over 99%, this improvement is not remarkable considering other existing heuristic solutions. What is important is that CNR can be easily extended to take advantage of the potential benefits of agent-based scheduling approaches. As an agent system, CNR can distribute computational requirements over several computer systems, include other solution methods at various points in of the rostering problem, and act as a real-time scheduling system. These benefits are not normally present in traditional centralized heuristic solutions and give CNR greater flexibility. Especially notable is that ability to act as a real-time scheduling system has tremendous potential for solving problems related to rostering, namely rerostering.

CHAPTER 3 ADDING AN ITERATED LOCAL SEARCH

The principle shortcoming of our CNR implementation is that it has no way of taking advantage of staffing demand slack. To solve this problem we use a framework of iterated local searches (ILS) to improve the final CNR solution. Our ILS framework works within the Auction Control Agent (ACA) and the Broker Agents (BA) of the CNR auction. The principle function of the ILS framework is to allow individual BAs to alter their schedules by moving work stints through stint swapping. A work stint is any series of consecutive days where a nurse is working a shift. An off stint is any series of consecutive days where a nurse is not working. The stint concept is adapted from Millar and Kiragu's network programming model (1998) and was adapted for our CNR algorithm.

The ILS framework allows a BA to select a work stint in its schedule and swap it with an off stint in another part of its schedule. For example, if a BA's schedule includes three consecutive day shifts starting on day four the BA may elect to swap the stint with a three day off stint starting on day seven. After the swap the BA will have an off stint starting on day four and a work stint starting on day seven.

3.1 AUCTION CONTROL AGENT ILS

The ILS framework operates a small control segment in the ACA (ACA-C) and a small ILS in each BA (BA-ILS). The ACA-C is responsible for controlling the ILS framework's convergence and for converting the final demand picture from the CNR auction into a staffing slack picture. Convergence is controlled through a set of variables that track the number of consecutive BA-ILSs that failed to find feasible schedule

improvements. Schedule improvement is measured using the utility function from Chapter 2 in Equation 2.1. Once every BA-ILS fails to find an improvement in succession, the ACA-C considers the ILS framework to be converged. A high-level summary of the ILS framework and how it is implemented in the CNR agents is depicted in Figure 3-1.

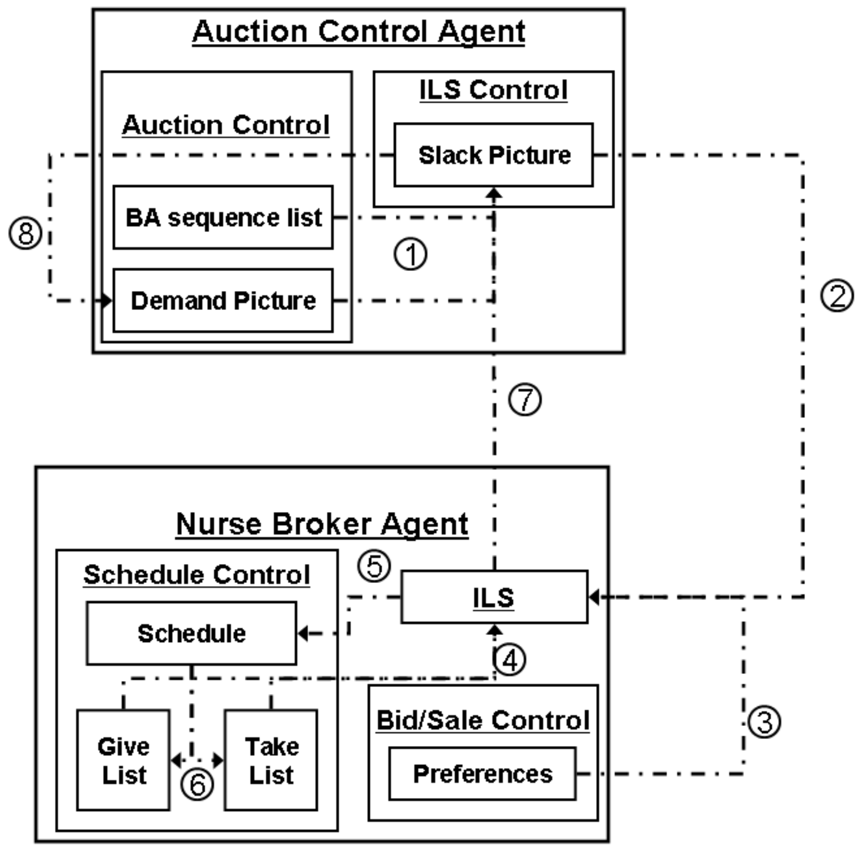

Figure 3-1: Depiction of how the parts of the BA and ACA interact with the new ILS framework. The ILS framework is constituted by additions to the existing auction control and broker agents. The 8 steps are as follows:

1. Seed the ILS control with the broker sequence and the final demand picture from the auction.

2. The ILS control instructs the BA to perform its ILS functions given the current slack picture.
3. The BA imports its preference structures from the auction to the ILS.
4. The BA uses its give and take lists to find the best possible stint swap.
5. The BA updates its schedule if needed.
6. The BA updates its give and take lists if needed.
7. The ACA-C updates its slack picture based on the BA's ILS results.
8. If the ILS framework has converged the ACA-C will use the slack picture to update the ACA's demand picture. If the framework has not converged the ACA-C will initiate step 2 for the next BA.

The staffing slack picture is stored in a set of two arrays, one for night shift slack and one for day shift slack. The slack picture, depicted in Figure 3-2, lets the ACA-C inform the BA-ILS functions of how many extra nurses of each training level are working each shift.

	Night Shifts					
Level	**0**	**1**	**2**	**3**	**…**	**27**
1	$SN_{1,0}$	$SN_{1,1}$	$SN_{1,2}$	$SN_{1,3}$		$SN_{1,27}$
2	$SN_{2,0}$	$SN_{2,1}$	$SN_{2,2}$	$SN_{2,3}$		$SN_{2,27}$
3	$SN_{3,0}$	$SN_{3,1}$	$SN_{3,2}$	$SN_{3,3}$		$SN_{3,27}$
	Day Shifts					
Level	**0**	**1**	**2**	**3**	**…**	**27**
1	$SD_{1,0}$	$SD_{1,1}$	$SD_{1,2}$	$SD_{1,3}$		$SD_{1,27}$
2	$SD_{2,0}$	$SD_{2,1}$	$SD_{2,2}$	$SD_{2,3}$		$SD_{2,27}$
3	$SD_{3,0}$	$SD_{3,1}$	$SD_{3,2}$	$SD_{3,3}$		$SD_{3,27}$

Figure 3-2: A representation of how the ACA-C stores the staffing slack picture. Each array contains the number of extra nurses of each level assigned to each day. One array is for the day shift and the second for the night shift.

The slack picture contains a set of variables that indicate extra nursing staff. SD_{rj} is the staffing slack of level r nurses during the day shift of day j. SN_{rj} is the staffing slack of level r nurses during the night shift of day j. The ILS uses these slack variables to control how it manipulates the existing nurse schedules so that they remain feasible.

When the BA-ILS is searching for a stint swap, staffing slack must be verified in the shifts that comprise the stint that the BA wants to have off. To verify that there is slack the BA-ILS must ensure that there are more than enough nurses at every training level working every shift in the stint. To do this the constraints in Equation 3.1 must be satisfied.

$$SN_{rj} \text{ and } SD_{rj} \geq 0 \quad \forall\, r \leq \text{current nurse's training level} \qquad (3.1)$$

The Algorithm for the ACA-C has three parts. First the slack picture is produced, second the BA-ILS functions are run, and third the results are integrated back into the primary ACA data structures for final output. The whole ACA-C algorithm is shown in pseudo code in Figure 3-3.

```
set failure count = 0;
determine staffing slack picture;
LOOP through all BAs until ILS failure count = number of BAs
        send current slack picture to current BA-ILS;
        IF BA-ILS failed to change current BA's schedule THEN
            increment failure count;
        ELSE
            set failure count to 0 and update slack picture;
        END IF
END LOOP
update ACA demand picture;
```

Figure 3-3: The ACA-C algorithm in pseudo code.

3.2 BROKER AGENT ILS

The BA-ILS runs as an extension of the BA. This allows it to have access to the BA's schedule and its preference data structures from the CNR auction. The preference data is used to determine what schedule manipulations are most advantageous to each nurse. With access to the BA's data, the BA-ILS works to alter the schedule by producing pair-wise stint swaps within schedule of the BA.

The BA-ILS algorithm uses the BA's give and take lists to search for stints to swap. The BA-ILS will only swap two stints when the swap produces schedules with higher utility values. This property of the BA-ILS ensures that the utility functions are strictly increasing in value and guarantees the whole ILS framework will converge. Of

course, since this is a heuristic method, there is no guarantee of global optimality. The BA-ILS algorithm is shown in Figure 3-4 in pseudo code.

```
LOOP through give list stints
    IF the current give stint has staffing slack for every shift THEN
        LOOP through take list stints
            IF current take stint size = current give stint size THEN
                perform hypothetical trade of give stint for take stint;
                IF trade is feasible and improves utility  THEN
                    store the trade and magnitude of utility improvement;
                END IF
            END IF
        END LOOP
    END IF
END LOOP
IF no hypothetical trades improved the schedule utility THEN
    report failure;
    end ILS;.
ELSE
    implement best hypothetical trade on BA's schedule;
    report success;
    end ILS;.
END IF
```

Figure 3-4: The BA-ILS algorithm in pseudo code.

The BA-ILS is designed to find and perform only one pair wise stint swap. This is an intentional design that allows the ACA-C code segment to give fair treatment to each BA. The ACA-C calls each BA-ILS sequentially until every BA-ILS consecutively fails to perform a swap. The fact that each BA-ILS performs only one swap at a time ensures that each BA is treated equally.

3.3 EXPERIMENTS

The CNR-ILS algorithm was tested based on nurse rostering at the Mike O'Callaghan Federal Hospital's Air Force Medical Surgical Unit (MOFH-AFMSU). This

inpatient ward has 20 registered nurses (RNs) who work in 12-hour shifts around the clock. The MOFH-AFMS ward manning rules are detailed in Appendix B.

CNR with the ILS framework (CNR-ILS) was tested on the same 20 random experimental runs and with the same settings used in Chapter 2. The performance of CNR-ILS is then compared to both the performance of the CNR auction without the ILS and the performance of an integer program (IP) adapted from Azaiez and Al Sharif (2005). This IP program was selected because it handles preferences in a relatively common manner. While there are other many other mathematical programs that we could have used, the Azaiez and Al Sharif model is representative of them and competitive computationally.

Let us turn our attention now to the performance characteristics of the original CNR model, the CNR-ILS model, and the IP model. We will measure the quality of the solutions produced three ways: the number of on-off-on patterns, the percentage of ROs satisfied, and the utility function values for each nurse. We will also compare the runtimes of the CNR and CNR-ILS models (both are orders of magnitude superior to the IP).

The way nurse preferences are handled in our comparison warrants some discussion. Our adaptation of Azaiez and Al Sharif's IP model considers fewer nurse preferences than our CNR and CNR-ILS models, but is pretty accurate overall. The IP includes ROs, balanced weekends off, and on-off-on patterns. The IP also includes the constraints necessary to satisfy MOFH-AFMSU manning rules. Table 3-1 is a comparison of how the three models consider nurse preferences. Unlike the IP, the CNR

and CNR-ILS models handles weekends differently and considers off stints of various lengths. This flexibility of CNR-based methods compared to IP is one of its advantages.

Table 3-1: Comparison of how the IP model, the CNR model, and CNR-ILS model handle various schedule preferences for the individual nurses.

CNR and CNR-ILS Models	**IP Model**
Nurses can indicate a preference for either weekends or weekdays off	Nurses must have at least half the weekends off
Nurses can request up to four days off	Nurses can request up to four days off
On-off-on patterns are penalized	On-off-on patterns are penalized
Nurses can indicate a preferred number of consecutive days off	
Working 5 days out of any 6 is penalized	

The 20 experimental runs using the adapted Azaiez and Al Sharif IP model were are solved on a 1.4GHz Centrino computer using COIN-CBC. Each IP developed for 20 nurses and a 28 day schedule includes 1120 integer variables and 560 continuous variables. The mathematical formulation is represented in equations 3-14.

$$Min \sum_{i \in I} \sum_{j \in J} \left(P_{ij} XD_{ij} + P_{ij} XN_{ij} \right) + \sum_{i \in I} C_1 d_{ij} \tag{3.3}$$

The objective function in Equation 3.3 includes a term for the preference impact of nurse i working either a night shift or day shift on day j. In this term P_{ij} is the preference impact of nurse i working on day j. The last term is a penalty for any on-off-on patterns in the schedule. The d_{ij} variable indicates that nurse i's schedule has an on-off-on pattern at the j^{th} day. I is the set of nurses and J is the set of days in the scheduling period. The decision variables are constrained as follows:

$$XD_{ij}, XN_{ij} \in \{0,1\}, \quad \forall i \in I, \quad \forall j \in J$$
$$d_{ij} \geq 0, \forall i \in I$$

The first five constraints (below) enforce workload requirements for the individual nurses. Equation 3.4 ensures that a nurse does not work more than 3 consecutive days. Equation 3.5 ensures that a nurse cannot have more than 4 consecutive days off. Equation 3.6 and 3.7 ensure that no nurse will work back-to-back shifts. Equation 8 ensures each nurse works their required number of shifts as indicated by variable $W_{i.}$

$$\sum_{i=n}^{n+3} XD_{ij} + \sum_{i=n}^{n+3} XN_{ij} < 4, \quad 0 \leq n \leq |I| - 3, \quad \forall j \in J \tag{3.4}$$
$$\sum_{i=n}^{n+4} XD_{ij} + \sum_{i=n}^{n+4} XN_{ij} > 0, \quad 0 \leq n \leq |I| - 4, \quad \forall j \in J > 0 \tag{3.5}$$
$$XD_{ij} + XN_{ij} \leq 1, \quad \forall i \in I, \forall j \in J \tag{3.6}$$
$$XD_{ij} + XN_{i-1j} \leq 1, \quad \forall i \in I, i \neq 0, \forall j \in J \tag{3.7}$$
$$\sum_{j \in J} \left(XD_{ij} + XN_{ij} \right) = W_i, \forall i \in I \tag{3.8}$$

Equations 3.9-3.12 are staffing constraints. Equations 3.16 and 3.17 ensure that the required numbers of nurses are assigned to each day and night shift. Equations 3.18 and 3.19 ensure that the required numbers of level 3 nurses are assigned to each day and night shift.

$$\sum_{j \in J} XD_{ij} \geq D_j, \forall i \in I \tag{3.9}$$
$$\sum_{j \in J} XN_{ij} \geq N_j, \forall i \in I \tag{3.10}$$
$$\sum_{j \in J_3} XD_{ij} \geq D3_j, \forall i \in I_3 \tag{3.11}$$
$$\sum_{j \in J_3} XN_{ij} \geq N3_j, \forall i \in I_3 \tag{3.12}$$

Equation 3.13 governs workloads for individual nurses around periods of vacation time. In this equation V_{ik} is the set of days surrounding vacation period k for nurse i. This set includes the two days preceding and two days following the vacation period. K is the set of vacation periods for a given nurse in the scheduling period. This constraint ensures that a nurse can only have off three of the four days surrounding a vacation.

$$\sum_{j \in V_{ik}} \left(XD_{ij} + XN_{ij} \right) > 0, \forall k \in K, \forall i \in I \tag{3.13}$$

The final constraint in Equation 3.14 ensures that every nurse has at least half of the weekend days off. E_J is the set of weekend days in the scheduling period. In this IP, the maximum number of weekend days a nurse can work is set to four because the scheduling periods at MOFH-AFMSU are 28 days long and include eight weekend days. Weekends for the night shift are Friday and Saturday nights.

$$\sum_{j \in E_J} \left(XD_{ij} + XN_{ij} \right) \leq 4, \ \forall i \in I \tag{3.14}$$

The three models (CNR, CNR-ILS and IP) are also compared to the current by-hand solution used at the Mike O'Callaghan Federal Hospital Air Force Medical Surgical Unit (MOFH-AFMSU). The study was performed in a double blind format. The nurses were presented four schedules each month in a random order. The source of each schedule, CNR, CNR-ILS, IP or by-hand method, were not divulged to the nurses. The comparisons included four months of testing over four 28-day scheduling periods.

Each month the nurses filled out a questionnaire to survey their preferences with respect to off duty stint lengths, weekends or weekdays off, and specific informal requests off (ROs). We then developed four schedules using CNR, CNR-ILS, the IP, and the MOFH-AFMSU by-hand method. The nurses were asked to rate each schedule with

respect to their level of satisfaction. Satisfaction levels were surveyed using an 11 point Likert scale anchored at -5 = very dissatisfied, 0 = indifferent, and 5 = very satisfied. The formats for the nurse satisfaction survey and the preference questionnaire are presented with the original CNR research [4].

The CNR and CNR-ILS algorithms started with an initial roster where each nurse is scheduled to work a 28 day period of alternating on and off days. Half the nurses start their schedules with an off day and half with an on day. This initial roster has five nurses scheduled to work each shift on each day.

3.4 RESULTS

The CNR-ILS algorithm performed very well over the 20 random experimental runs. Table 3-2 presents the key performance characteristics of the algorithm where there are 20 nurses, 10 of whom are assigned to the day shift and 10 to the night shift. In this table the minimum demand for nurses is four per shift. Table 3-3 represents the data for the IP model over the same 20 runs with a demand of four nurses per shift. Table 3-4 represents the data for CNR-ILS where the demand is three nurses per shift.

Table 3-2: Performance characteristics of the CNR-ILS algorithm over 20 experimental runs where the demand for nurses is four per shift. The data points include the average for all nurses per run and the standard deviation for all nurses per run. The last row is the average of the data points in its respective column.

Run	**Avg. Num On-Off-On Patterns**	**Std. Dev. On-Off-On Patterns**	**Avg. Ratio of ROs Granted**	**Std. Dev. Ratio of ROs Granted**	**Run Time (Seconds)**
1	0	0	1	0	37

Run	Avg. Num On-Off-On Patterns	Std. Dev. On-Off-On Patterns	Avg. Ratio of ROs Granted	Std. Dev. Ratio of ROs Granted	Run Time (Seconds)
2	0	0	1	0	36
3	0.15	0.37	1	0	36
4	0	0	0.95	0.22	36
5	0	0	1	0	44
6	0	0	1	0	38
7	0	0	1	0	33
8	0	0	1	0	40
9	0.05	0.22	1	0	42
10	0.05	0.22	0.93	0.24	37
11	0.1	0.31	1	0	42
12	0.15	0.49	0.98	0.07	35
13	0.05	0.22	1	0	36
14	0	0	1	0	42
15	0.05	0.22	1	0	40
16	0.05	0.22	1	0	36
17	0	0	0.98	0.11	37
18	0	0	0.9	0.31	39
19	0.05	0.22	0.93	0.24	38
20	0	0	0.98	0.11	31
Over All Runs	0.04	0.13	0.98	0.07	37.8

CNR-ILS leverages the staffing demand slack for each shift and produces better results than CNR alone. The CNR-ILS framework significantly improves our original CNR algorithm with respect to the number of on-off-on patterns but not the average ratio of ROs satisfied. Running the ILS framework adds only 1-2 seconds to the runtime of the CNR algorithm.

Adding the ILS to the CNR algorithm reduced the average number of on-off-on patterns by 90% over the 20 random experimental runs. The standard deviation of the on-off-on patterns was reduced by 80%. A reduced standard deviation assures us that we are not getting erratic solutions. It is no guarantee of optimum, but an assurance

nevertheless. When the nursing demand is reduced to three nurses per shift from four nurses per shift the reduction in on-off-on patterns was 95% and the standard deviation was 87%. These improvements are reflected in a p-value of 0.00 on the equality of means at 95% confidence.

The improvements were not as large for the mean ratio of ROs satisfied per run. CNR-ILS had a satisfaction rate 1% higher than CNR and the standard deviation was 36% less when there are four nurses required per shift. When there we only three nurses required per shift the satisfaction rate was 2% higher and the standard deviation was 45% less. This indicates that the CNR-ILS algorithm was more consistent in granting ROs compared to the CNR algorithm. A test on the equality of means returned a p-value of 0.07 which does not indicate a statistical difference in the means.

Table 3-3: Performance characteristics of the IP model over 20 experimental runs where the demand for nurses is four per shift. The data points include the average for all nurses per run and the standard deviation for all nurses per run. The last row is the average of the data points in its respective column. The runtime for this model is on the order of hours to days.

Run	**Avg. Num On-Off-On Patterns**	**Std. Dev. On-Off-On Patterns**	**Avg. Ratio of ROs Granted**	**Std. Dev. Ratio of ROs Granted**
1	0	0	1	0
2	0	0	0.98	0.11
3	0	0	0.98	0.11
4	0	0	0.95	0.22
5	0	0	1	0
6	0	0	0.98	0.11
7	0	0	1	0

Run	Avg. Num On-Off-On Patterns	Std. Dev. On-Off-On Patterns	Avg. Ratio of ROs Granted	Std. Dev. Ratio of ROs Granted
8	0	0	1	0
9	0	0	0.95	0.15
10	0.05	0.22	1	0
11	0.05	0.22	1	0
12	0.1	0.45	0.95	0.22
13	0.05	0.22	1	0
14	0	0	1	0
15	0	0	1	0
16	0.05	0.22	1	0
17	0	0	1	0
18	0	0	1	0
19	0.05	0.22	0.95	0.22
20	0	0	1	0
Over All Runs	0.02	0.08	0.99	0.06

The CNR-ILS algorithm performs comparably to the IP model with respect to the ratio of ROs granted and the number of on-off-on patterns at the 95% confidence level. The run time of the IP model is on the order of hours rather than seconds.

The difference between the mean ratio of ROs granted per run by the IP model and the CNR-ILS model is not statistically significant. A test of equality returns a p-value of 0.29. The IP was a 1% improvement over the CNR-ILS algorithm with respect to the mean. The difference in the standard deviation of the IP was a 14% improvement over the CNR-ILS algorithm. The IP model's mean ratio of ROs satisfied was statistically different from the original CNR model with a p-value of 0.02.

When compared to the IP, the CNR-ILS algorithm shows an increase in the average number of on-off-on patterns per run of 50% and an increase in the standard deviation of 38%. Because of the small number of on-off-on patterns allowed by both

algorithms, the difference in the means is not statistically significant. A test of equality returns a p-value of 0.06.

The ILS algorithm in the CNR-ILS framework ensures the utility functions are strictly increasing. As a result, CNR-ILS favors an improved outcome when minimum staffing demand is reduced. With any reduction in staffing demand the algorithm is afforded more flexibility and has a larger solution space. The key benchmarks for the CNR-ILS with a demand of three nurses per shift are shown in Table 3-4. When the staffing demand is reduced to three nurses per shift for the CNR-ILS model there is not a statistical improvement with respect to the mean ratio of ROs granted and the mean number of on-off-on patterns per run. The p-values for the tests of equality are 0.07 and 0.09 respectively.

Table 3-4: Performance characteristics of the CNR-ILS algorithm over 20 experimental runs where the demand for nurses is three per shift. The data points include the average for all nurses per run and the standard deviation for all nurses per run. The last row is the average of the data points in its respective column.

Run	Avg. Num On-Off-On Patterns	Std. Dev. On-Off-On Patterns	Avg. Ratio of ROs Granted	Std Dev Ratio of ROs Granted	Run Time (Seconds)
1	0	0	1	0	37
2	0	0	1	0	37
3	0	0	1	0	37
4	0	0	0.95	0.22	36
5	0	0	1	0	45
6	0	0	1	0	38
7	0	0	1	0	34
8	0	0	1	0	40
9	0	0	1	0	42

Run	Avg. Num On-Off-On Patterns	Std. Dev. On-Off-On Patterns	Avg. Ratio of ROs Granted	Std Dev Ratio of ROs Granted	Run Time (Seconds)
10	0.05	0.22	0.95	0.22	40
11	0.05	0.22	1	0	43
12	0.1	0.45	1	0	34
13	0.05	0.22	1	0	36
14	0	0	1	0	41
15	0	0	1	0	40
16	0.05	0.22	1	0	37
17	0	0	0.98	0.11	37
18	0	0	0.9	0.31	39
19	0.05	0.22	0.95	0.22	39
20	0	0	0.98	0.11	32
Over All Runs	0.02	0.08	0.99	0.07	38.2

The utility values for each nurse were compared in a pair wise manner. Tests were performed over each experimental run and over all the observations from all 20 runs. Table 3-5 lists the upper and lower bounds of the 95% confidence intervals for the average difference in the utility values for each nurse from the CNR and the IP algorithms, the CNR-ILS and IP algorithms and the CNR-ILS algorithm when staffing demand is three and four nurses per shift. In Table 3-5 any confidence interval that includes zero is bolded. These bolded confidence intervals indicate no statistical evidence that the average difference in the utility values is not zero.

Table 3-5: Confidence intervals on the difference of the utility values for the nurses in each run and for all the runs at the 95% level. The entries in bold include zero and are not statistically different. The differences used to develop these confidence intervals are

defined by taking the utility values from the model on the left of the column title and subtracting the utility values of the model on the right.

	CNR - IP		**CNR-ILS - IP**		**CNR-ILS (Demand = 3) – CNR-ILS**	
Run	**Lower**	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
1	-1.35	-0.28	3.33	4.19	0.69	1.01
2	-6.37	-3.16	5	5.73	0.74	1.07
3	-7.77	-4.88	3.26	4.58	1.73	2.31
4	-4.32	-2.12	4.46	5.45	0.67	0.88
5	**-0.9**	**0.51**	4.07	4.92	0.5	0.8
6	**-0.23**	**1.74**	5.25	5.86	0.29	0.42
7	**-1.28**	**0.58**	4.94	5.84	0.43	0.68
8	**-1.06**	**0.14**	3.84	4.86	0.54	0.91
9	-4.91	-3.07	2.68	3.6	1.38	2
10	-4.38	-2.1	3.02	4.02	1.03	1.49
11	-4.17	-2.58	2.06	3.01	0.61	1.11
12	-6.16	-4.09	1.24	2.73	1.78	2.99
13	-2.94	-1.64	3.41	4.12	0.3	0.46
14	-1.2	-0.06	4.7	5.36	0.41	0.58
15	-5.19	-3.29	4.75	5.81	0.66	1.01
16	-5.3	-2.93	2.47	3.35	0.88	1.28
17	-1.56	-0.03	4.32	5.69	0.63	0.85
18	-5.4	-3.71	0.57	1.9	0.1	0.17
19	-9.95	-7.14	**-1.21**	**0.79**	3.19	4.86
20	-5.78	-3.8	3.64	5.07	0.52	0.72
Over All Runs	-4.05	-2.06	3.27	4.38	0.78	1.35

When comparing the original CNR algorithm to the IP it is not surprising that the difference in the utility values is significant. The original CNR algorithm assumes minimal manning and therefore has a reduced solution space. When the ILS is added to create the CNR-ILS algorithm the difference compared to the IP is again statistically significant, however, the CNR-ILS algorithm provides the better utility values. The reason this heuristic can outperform an IP solved to optimality is that the heuristic is

capable of easily considering a larger set of preferences. In this study the CNR-ILS algorithm included the preferred length of off-duty stints.

From the perspective of the users, CNR-ILS outperformed CNR, the IP, and the MOFH-AFMSU by-hand methods in our double blind study. Table 3-6 reports the key survey metrics for all four scheduling methods. The first metric is the percentage of nurse ratings that indicated dissatisfaction. The next five metrics are the average nurse ratings where -5 = Very Dissatisfied and 5 = Very Satisfied. The final 5 metrics are the standard deviation of the nurse ratings.

Table 3-6: Comparison of the four scheduling methods with respect to nurse satisfaction ratings. These rating were obtained via a double blind study over four months.

	CNR	**CNR-ILS**	**IP**	**MOFH**
% Rating That Are Negative	33%	10%	21%	25%
Avg. Rating Month 1	0.7	3.45	1.7	2.9
Avg. Rating Month 2	0.71	2.5	1.79	1.36
Avg. Rating Month 3	2.64	3.27	1.64	1
Avg. Rating Month 4	1	2.08	1.23	1.85
Avg. Rating All Months	1.23	2.76	1.58	1.73
Rating Std Dev Month 1	3.59	1.07	3.33	3.03
Rating Std Dev Month 2	2.81	1.91	2.78	2.65
Rating Std Dev Month 3	3.11	2	1.75	2.49
Rating Std Dev Month 4	2.68	2.33	3.22	2.91
Rating Std Dev All Months	3.03	1.94	2.76	2.77

Table 3-6 shows that CNR has fewer bad ratings, less variation in the level of nurse satisfaction, and a higher average rating. These results indicate that CNR is more consistent and fairer than the other three methods. CNR, the IP and the by-hand method perform comparably to each other with higher standard deviations, more negative ratings

and lower averages. The averages of these three methods all fall with in a 0.5 point range while CNR-ILS is 1.03 points over that range.

The results from Table 3-6 are supported by the Paired T-Test results in Table 3-7. Table 3-7 shows that CNR, the IP, and the by-hand method are not statistically different with $\alpha=0.05$. Only CNR-ILS stands out from all the models in the double blind study. CNR-ILS' average ratings are statistically higher than the other three methods.

Table 3-7: The p-values from a paired two-tailed equality of means test. This test was performed on nurse satisfaction ratings from all four test months. The results show the CNR-ILS is prefered to the other three methods and that CNR is comparable to the MOFH by-hand method and the IP method.

	P-Value
CNR-ILS ≠ CNR	0.002
CNR-ILS ≠ IP	0.017
CNR-ILS ≠ MOFH	0.033
MOFH ≠ CNR	0.393
MOFH ≠ IP	0.756
IP ≠ CNR	0.519

Figure 3-5 graphically represents the monthly average ratings for all four scheduling methods. This figure highlights the second and fourth month because the staffing levels during those two months were abnormal. During the second month staffing was unusually low due to training assignments and vacations. During this month there was a drop in the average ratings.

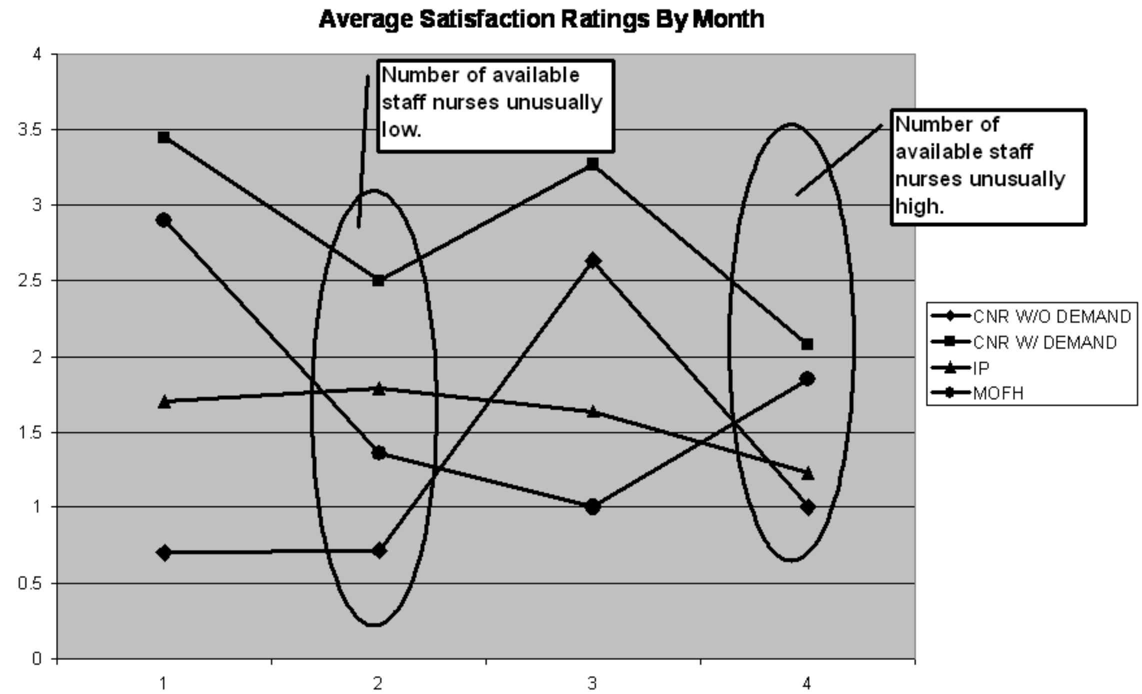

Figure 3-5: Depiction of the average satisfaction ratings for the four scheduling methods over the test months. Month two and four are circled as examples of an unusually hard and unusually easy staffing month respectively. In month two several nurses where not available resulting in reduced available staffing numbers. In month four several new nurses joined the MOFH-AFMSU to replace the some departing nurses. Since the departing nurses had not left yet the available staffing numbers where unusually high.

During the fourth month staffing was unusually high. The MOFH-AFMSU received several new nurses that were supposed to replace several departing nurses. During this month the departing nurses had not yet left the unit. As a result both the new nurses and the departing nurses were available during this scheduling period. The high level of available staffing made the scheduling problem for the last month easier. This easier problem resulted in smaller differentiation between the schedules used in the

double blind study. As a result the average ratings for the four methods were closer and as a group lower.

The fourth month highlights one of the problems inherent in surveying nurse satisfaction over several schedules every month – framing. When rating each schedule the nurses cannot forget schedules they already rated and they therefore rate each schedule within the context of the other three. Several times this framing effect was evident in nurse satisfaction surveys. For example in one survey a nurse rated the first schedule a 2 then after rating the second schedule a 2 the nurse crossed out the first rating and re-rated the first schedule a 1. By the end of the survey the rating for the first schedule was changed from 2 to -1.

3.5 DISSCUSSION

While CNR-ILS is a good heuristic, it can be extended or improved in several ways. First, the CNR auction can be improved to include bids that are not the same size as the sale item. For example if a nurse wants to sell a three day work stint another can bid with three one-day work stints or a two-day and a one-day work stint. This would increase the algorithm's flexibility to move through the solution space.

Second, the auction mechanism can be altered to handle staffing slack. This can be done by hybridizing the ILS stint swapping mechanism and the auction mechanism. In this hybridized auction a nurse could sell a three-day work stint by trading it for a two day work stint and swapping the remaining one-day work stint with a one-day off stint from somewhere else in the selling nurse's schedule.

Third, the development of a specific cost minimization model to use as input to the CNR-ILS algorithm could be valuable. While CNR-ILS was tested for four months

and did perform well we did not address the positive or negative effects of using different input rosters.

CHAPTER 4 COMPETITVE NURSE REROSTERING

The rerostering of nurses occurs when there is a disruption to a roster that requires its modification. This study applies disruptions to rosters produced by CNR-ILS. Each disruption is to a single nurse's schedule and renders that nurse unavailable for up to three consecutive days.

CNR-ILS uses a work shift trading mechanism that relied on the swapping of work stints between different nurses. While this mechanism has been shown to work well for rostering nurses, a new more flexible trading mechanism is used for rerostering. In this new trading mechanism the selling nurse still sells a single work stint. However, when bidding, nurses can offer any combination of work shifts that have the same number of shifts that are in the stint being sold. This new, more flexible trading mechanism is called advanced trading (AT). AT allows nurses to bid with a variety of shorter stints in an effort to increase the likelihood of a trade. This added flexibility helps minimize the negative effect of any trade on a bidding nurse's schedule.

4.1 ADVANCED SHIFT TRADING

AT requires a redesign of the currency list used in the original CNR auction. In the original CNR algorithm, the currency list represented all the work stints that a nurse would accept in exchange for the work stint she was selling. The currency list is sorted in descending order with the first stint was the one that results in the most favorable trade for the selling nurse. This currency list only includes stints that are equal in size to the stint that is for sale. When modified for AT, the currency list contains a sequence of shift

sets where each set has the same number of work shifts as is in the work stint that is for sale. The redesign of the currency list is depicted in Figure 4-1.

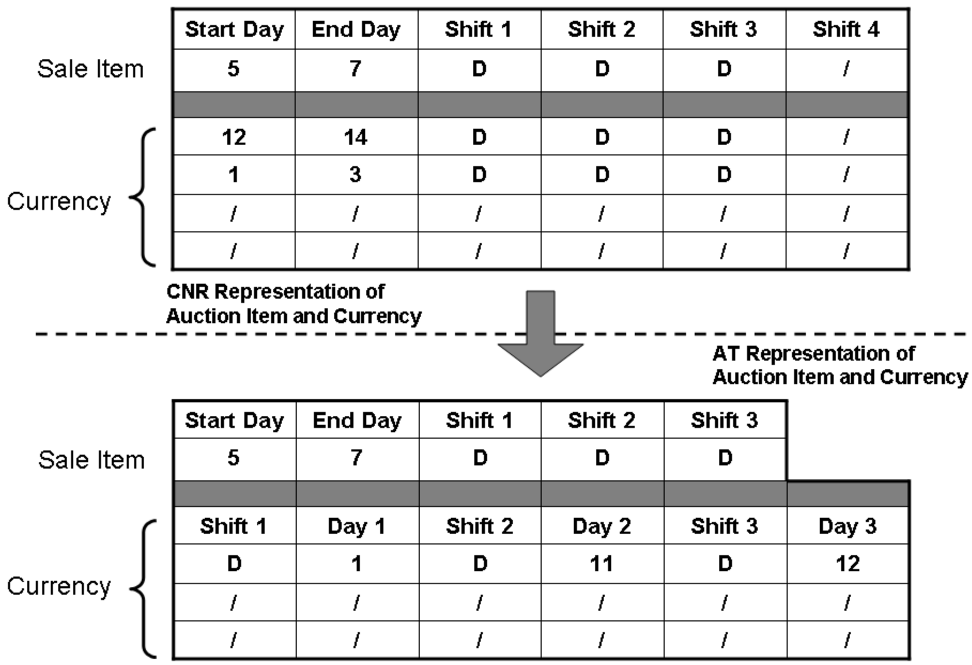

Figure 4-1: This figure depicts the modification done to the CNR auction item for AT. There are no changes to the sale item but the currency list simply contains any work shifts, not just consecutive shifts. In this image the selling nurse is trying to sell the day shifts on days 5, 6 and 7. The seller is asking for the day shifts on days 1, 11, and 12 in exchange.

The CNR auction mechanism is more powerful when it is given the added flexibility of using AT. This added flexibility allows the selling and bidding nurses to consider all the trades possible in the original CNR plus any additional trades that result from bidding with nonconsecutive shifts. This larger set of possible trades means that any single trade will be at least as good as the original CNR.

The AT trading mechanism was tested in the CNR and CNR-ILS algorithms using the same random experiments from our previous work (Chiaramonte 2008). In our previous work the CNR and CNR-ILS algorithms were tested using random experiments where the bid threshold (B) was set to 10, the single off penalty (C_1) was set to 10, the balance penalty (C_2) was set to 5, and the maximum number of iterations was set to 250. A design of experiments performed during our previous research on the CNR algorithm determined that these settings produced the most favorable results. The same experiments were rerun to test these algorithms when the AT trading mechanism is incorporated.

4.1.1 ROSTERING RESULTS

The CNR and CNR-ILS algorithms were compared with and without the AT mechanism is in place. The key performance metrics that were recorded are the number of single days off, the ratio of informal request offs granted, and the utility measures of the individual nurses. Table 4-1 shows the p-values from the tests of equality for CNR and CNR-ILS compared to the same algorithms when AT is used.

Table 4-1: This table shows the p-values for the equality of means tests on key performance indicators. The p-values show that using advanced trading improves the CNR algorithm with respect to the number of single days off.

Algorithm Comparison	**Single Days Off**	**Ratio of Request Offs (ROs) Granted**	**Individual Nurse Utility**
CNR	0.00	0.11	0.18
CNR-ILS	0.25	0.10	0.39

Implementing AT for CNR and CNR-ILS does not improve the overall performance of the rerostering algorithm. Only the improvement for the average number of single days off when using CNR was statistically significant. Conversely when CNR-ILS had AT implemented the observed difference in averages were not statistically significant.

While it is not apparent that AT improves the overall CNR or CNR-ILS algorithm AT does improve the individual trades. When AT is implemented the average number of trades per nurse for a 28-day schedule with 20 nurses decreases from 45.7 to 30.1. This difference is statistically significant with a p-value of 0.00. The added complexity of the AT mechanism increases the run time of the CNR algorithm to an average of 196 seconds from 37 seconds.

4.2 REROSTERING

Similar to CNR and CNR-ILS, CNRR relies on one-to-one shift trading. This one-to-one ratio ensures that any CNRR solution conforms to Moz and Vaz Pato's definition of optimality (2003). They defined optimal as any solution that minimizes the differences between the original and new roster. Unlike their solutions, CNRR also considers preferences while solving for this optimality.

CNRR modifies the utility function that is used in CNR and CNR-ILS. The modification allows nurses to work the opposite shift from which they are assigned. Thus the exchange of day and night shifts, while not desirable, is possible in CNRR. The interchanging of day and night shifts is controlled by the addition of a new term to the utility function. This term, detailed in Equation 4.1, assigns a penalty (C_3) to minimize

these exchanges. In this term X_j indicates a day were the nurse is assigned to the wrong shift.

$$-\sum_{j \in J} C_3 X_j \tag{4.1}$$

In CNRR, disruptions are treated as request offs (ROs) and can be no longer than three days. Limiting disruption lengths helps enforce a staffing rule from our test hospital that requires nurses to take vacation time when they will be off duty for more than three consecutive days. When adding a disruption, a RO is added to the nurse's preference structure for each day that is disrupted. Each single RO is added with a very large preference weighting. Figure 4-2 depicts a disruption added to a nurse's preference list. In this figure the nurse experiences a three-day disruption in her schedule starting on day one.

Start Day	End Day	Weight
		R_1
		R_2
		R_3
		R_4
1	1	M
2	2	M
3	3	M

Figure 4-2: This image depicts the addition of a three day disruption to the preference list of a nurse. Each day in the disruption is added as a single day stint with a large weight M.

CNRR uses a three stage approach to minimize the impact that rerostering has on nurse preferences. These stages are Schedule Improvement, Impact Isolation, and Impact

Minimization. During the first and last stage, CNRR tries to fix a disruption by using the CNR auction mechanism with Advanced Trading (AT). During the second stage, CNRR attempts to fix a disruption using the ILS framework introduced in the CNR-ILS algorithm. In every stage CNRR attempts to trade the entire disrupted work stint in one trade. Failing to do this it will attempt to trade away the disruption in the least number of trades. Figure 4-3 depicts the order that CNRR will use when attempting to solve a three-day disruption that starts on day one.

Disruption

	Start	**End**	**S1**	**S2**	**S3**	**S4**
	1	3	D	D	D	/

Stint that disrupted nurse will attempt to sell and the order in which the sales will be attempted

Sale Order	**Start**	**End**	**S1**	**S2**	**S3**	**S4**
1	1	3	D	D	D	/
2	1	2	D	D	/	/
3	2	3	D	D	/	/
4	1	1	D	/	/	/
5	2	2	D	/	/	/
6	3	3	D	/	/	/

Figure 4-3: This image depicts the set of stints that are developed from a three day disruption. The nurse experiencing the disruption will attempt to sell these stints at auction in the order shown. In this image the nurse was assigned to the day shift for all three days in the disruption.

The first stage is called Schedule Improvement (SI) because the goal of CNRR is to use the CNR auction to improve the utility of the bidding nurses. As a result of this

goal, the bid threshold in the CNR auction is set to zero. The bid threshold determines how much of a utility loss the bidding nurse is willing to accept in any auction sale. The SI stage's algorithm is depicted in Figure 4-4.

```
Set C as the set of stints comprise the disruption;
Sort C so that largest stint is first;
LOOP through all stints in C
        Set bid threshold to 0.0;
        Execute CNR Auction iteration on the stint with Advanced Trading;
        IF shifts were traded THEN
            Remove any stints in C that had at a shift satisfied by the shift trade;
        END IF
END LOOP
IF C is empty THEN
        End CNRR;
ELSE
        Execute Impact Isolation Stage;
END IF
```

Figure 4-4: Pseudo code for the schedule improvement stage of CNRR. This stage is a CNR auction with AT where the bid threshold is set to zero. In this stage the set of stints that comprise the disruption are offered at auction.

When CNRR fails to satisfy an entire disruption in the SI stage the algorithm advances to the Impact Isolation (II) stage. This second stage accepts that there are no shift trades between nurses that can improve the utility of bidding nurses and attempts to ensure that any negative effects are isolated to the disrupted nurse. To do this, CNRR uses the CNR-ILS mechanism. In this stage, the ILS uses staffing slack to move work assignments from disrupted days to other days in the nurse's schedule. This stage of CNRR is only effective when the days involved in the disruption are not minimally staffed. The algorithm for the Impact Isolation stage is depicted in Figure 4-5.

```
LOOP Through stints in set C
Execute ILS framework in disrupted nurse's BA on the current stint;
        IF shifts were swapped THEN
           Remove any stints in C that had at a shift satisfied by the shift trade;
        END IF
END LOOP
IF C is empty THEN
        End CNRR;
ELSE
        Execute Impact Minimization Stage;
END IF
```

Figure 4-5: Pseudo code for the impact isolation stage of CNRR. This stage uses the CNR-ILS mechanism to attempt to satisfy the disruption by using staffing slack to shift the stints within the nurse's schedule. The ILS is run on all the stints in the set C that are not satisfied by stage 1 of CNRR.

When CNRR fails to fully satisfy a disruption during the SI and II stages, the algorithm advances to the Impact Minimization (IM) stage. This stage uses the same CNR auction mechanism with AT that is used in SI. Unlike the first stage, this stage uses a linear rollback (LRB) to allow stint trades that negatively impact bidding nurse utilities. LRB is used to minimize these negative impacts.

$$B_T = N * K \qquad (4.2)$$

LRB is a linear modification to the bid threshold. The bid threshold dictates how much utility a bidding nurse can lose on a single trade in a CNR auction. LRB, defined in Equation 4.2, slowly increases the bid threshold with each iteration of the IM stage. In this equation B_T is the bid threshold, N is the iteration number, and K is the iteration increment. The iteration increment determines how quickly the algorithm increases bid flexibility. Figure 4-6 depicts the IM stage in pseudo code where K is set to 0.05

```
Set K=0.05;
Set N=0;
LOOP from 1 to max iterations (MI)
        Set N=N+1;
        Set bid threshold to K*N;
        LOOP through all stints in C
            Execute CNR Auction iteration on the current stint with Advanced
                        Trading;
            IF shifts were traded THEN
                        Remove stints from C that had have any satisfied shifts;
            END IF
        END LOOP
        IF C is empty THEN
            end loop;
        END IF
END LOOP
IF C is empty THEN
        End CNRR;
ELSE
        Indicate CNRR Failed;
        End CNRR;
END IF
```

Figure 4-6: Pseudo code for the impact minimization stage of CNRR. This stage uses the CNR auction mechanism with Advanced Trading. In this stage the bid threshold is governed by linear rollback (LRB) which increases the bid threshold in a linear fashion until the CNRR algorithm succeeds or fails.

4.3 RESULTS

The rerostering experiments were run using the staffing rules of an actual hospital's medical surgical ward. These rules are detailed in Appendix B. CNRR cannot alter any initial roster if it results in a violation of the hard constraints listed in this table. The soft constraints reflect guidelines that the medical surgical ward tries to satisfy but are not required. CNRR was tested on initial rosters that were 28 days long and included 20 nurses. All of these initial rosters were generated from the output of the random CNR-ILS experiments from our previous research.

CNRR was tested using three sets of 30 experimental runs each. The settings for the CNRR test sets are detailed in Table 4-2. The first two test sets are designed to simulate a situation where a nursing roster, designed well before the work period covered, is disrupted in advance by some event.

The first two experimental sets use the same 30 runs. Each experimental run induces between one and five disruptions. Each disruption is between one and three days long. These sets of experiments treat paritially and fully satisfied ROs differently. In the first set of experiments, CNRR will not allow a shift trade that gives a nurse a work assignment during a day that is part of a RO. This trade restiriction is consistent with our prevous CNR and CNR-ILS research. The second set of experiments lifts this restriction.

The third set of experiments are designed to test CNRR when disruptions are realized during a scheduling period that has already begun. In this experimental set, nurses are only allowed to change their roster from day 14 onward.

The third experimental set does not use the same 30 runs as the first two sets. In the third set, each run consists of only one or two disruptions that start during the third week of the roster (between day 14 and 21). Like the first set of experiments, the third set handles partially and fully satisfied ROs the same way as our previous CNR and CNR-ILS research.

The CNR auction settings were determined through the use of a designed experiment in our previous research. The selection of the LRB settings were determined by comparing two factors, the effect on runtime and effectiveness. We selected a K of 0.05 because at this size it had little impact on runtime. Furthermore using a K smaller than 0.05 did not improve results When K was increased larger impacts were seen on

nurse preferences. The maximum number of iterations used for the LRB was set to 1000. This setting was selected because there were no experimental runs where the algorithm solved a disruption after 1000 iterations. The largest number of iterations seen was 901.

Using these settings, CNRR almost always solved disruptions in under 5 seconds. In the worst case scenario, when the algorithm failed to solve the disruption and ran through all 1000 LRB iterations, CNRR completed in approximately 15 seconds.

Table 4-2: This table is a summary of the three experimental sets used to test CNRR. Each set had 30 runs, and used the same CNR auction and LRB settings. In this table B_T is the bid threshold, C_s is the single day off penalty, C_B is the schedule balance penalty, C_w is the wrong shift penalty, K is the LRB iteration increment value, and MI is the maximum number of LRB iterations.

	Test Set 1	**Test Set 2**	**Test Set 3**
Start Day	0	0	14
Disruptions Per Run	1-5	1-5	1-2
Disruption Characteristics	1-3 days long	1-3 days long	1-3 days long Starting between day 14 and 21
CNR Auction Settings	B_T=LRB C_s=10 C_B=5 C_w=20	B_T=LRB C_s=10 C_B=5 C_w=20	B_T=LRB C_s=10 C_B=5 C_w=20
LRB Settings	K=0.05 MI=1000	K=0.05 MI=1000	K=0.05 MI=1000
RO Handling	Cannot Trade	Can Trade	Cannot Trade
Total Number of Disruptions	87	87	47

When testing CNRR each disruption is classified in three ways: the size of the disruption in days, the CNRR phase in which the disruption was finally satisfied, and

whether or not the disruption has a feasible solution with respect to the staffing rules. The first classification breaks down how well the CNRR algorithm handles disruptions of one, two or three days.

The second classification indicates the phase in which the disruption was satisfied. A three day disruption that was finally satisfied during the Impact Minimization (IM) phase may have had a day satisfied in the Schedule Improvement (SI) phase and a day satisfied in the Impact Isolation (II) phase.

The third classification indicates whether or not it is possible for CNRR to solve the disruption given the staffing rules. When testing, CNRR experienced two conditions that prevented the algorithm from finding a solution. The first is a condition that requires too many days off in a row and the second is one that requires too many days on in a row. These two conditions are depicted in Figure 4-7.

Table 4-3 details the ratios of disruptions solved by CNRR in the first experimental set. In this set there were 87 total disruptions over 30 runs. Of these 87 disruptions 81 were solvable while six were not. Five of the unsolvable disruptions were the result of Condition One while only one disruption was the result of Condition Two.

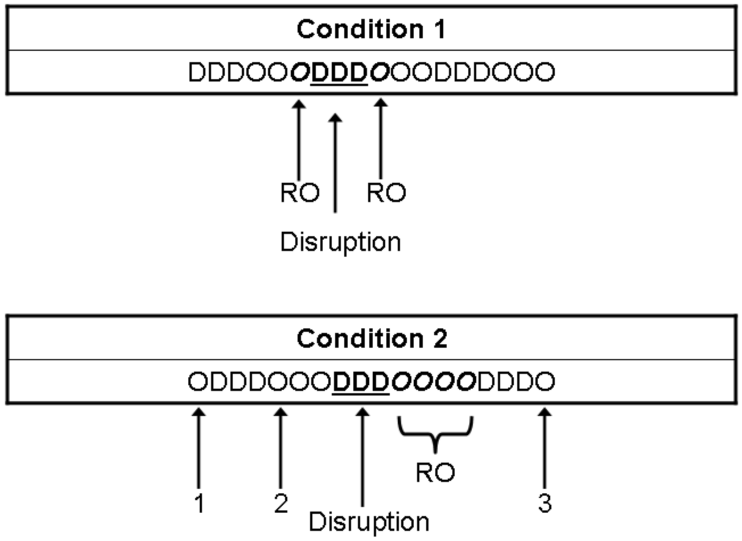

Figure 4-7: This figure depicts the two primary conditions where it is impossible for CNRR to solve disruptions. In this figure underlined 'D' indicates day shifts that are disrupted. Any bold italicized 'O' indicates a day off that is part of a RO. The first condition exists when a disruption is surrounded by ROs. Since CNRR will not allow trades where nurses take on work during part of a RO, solving the disruption will require more days off in a row than is allowed. The second condition exists when satisfying the disruption requires more than three days on in a row. In this condition at least one of the days labeled '1', '2', or '3' would need to be traded for a work shift to satisfy the disruption. This would result in four consecutive workdays.

The first two rows in Table 4-3 detail the overall results for CNRR while the next six rows show the results by algorithm phase. Every row titled with "possible" considerers only those disruptions that did not fall into Condition One or Condition Two. The columns break down ratios solved by the size of the disruption with the last column indicating the ratio solved over all disruptions.

CNRR solved 91% of the disruptions in the first experimental set. It was 98% when only solvable disruptions were considered. 2% of the disruptions were solved in the Schedule Improvement phase, 54% of the disruptions were solved during the Impact Isolation phase, and 38% of the disruptions required CNRR to enter the Impact Minimization phase. One third of the disruptions solved during the Impact Minimization phase required only one LRB iteration. One iteration equates to a utility impact that is negligible. Only 39% of the disruptions solved during the last phase required significant utility impact (at least 100 LRB iterations or 5 utility points).

Table 4-3: This table presents the ratio of the disruptions solved over 30 random experiments where each experiment had between one and five disruptions applied to a 28 day schedule. Disruptions varied in size from one day to three consecutive days. There were a total of 87 disruptions and 81 of those were solvable. Of the 87 disruptions 62% were one day long, 29% were two days long and 9% were three days long. The first two rows present the ratio solved for the entire algorithm while the next six rows show the ratio solved by each algorithm stage. Rows that indicate "possible" consider only those disruptions that do have feasible solutions.

	1 Day	**2 Day**	**3 Day**	**Total**
Ratio Solved	0.98	0.80	0.75	0.91
Ratio Possible Solved	1.00	0.95	0.86	0.98
Ratio SI	0.04	0.00	0.00	0.02
Ratio Possible SI	0.04	0.00	0.00	0.02
Ratio II	0.67	0.28	0.13	0.51
Ratio Possible II	0.68	0.33	0.14	0.54
Ratio IM	0.28	0.52	0.63	0.38
Ratio Possible IM	0.28	0.62	0.71	0.41

Table 4-4 displays the results of the second experimental set when CNRR is modified to allow nurses to trade days that are part of ROs. This table is organized in the same way as Table 4.4. When CNRR is allowed to trade ROs the algorithm solved 99% of the disruptions with no disruptions being impossible to solve. Similar to the results in Table 4.4, Table 4.5 shows that most disruptions were solved during the Impact Isolation phase. Impact Minimization solved 44% of the disruptions with 29% of those having negligible impact and 58% having little impact (less than 100 LRB iterations). Only 13% required at least 100 LRB iterations.

Table 4-4: This table presents data from the same experiments as Table 4-3. In this table the nurses were willing to give up days that were part of a request off to satisfy disruptions. There were a total of 87 disruptions and all 87 were solvable. The first two rows present the total solved while the next six rows show the ratio solved by each algorithm stage.

	1 Day	**2 Day**	**3 Day**	**Total**
Ratio Solved	1.00	1.00	0.88	0.99
Ratio SI	0.04	0.00	0.00	0.02
Ratio II	0.67	0.36	0.13	0.53
Ratio IM	0.30	0.64	0.75	0.44

The third experimental set consists of 30 experimental runs totaling 47 disruptions. The results are displayed in Table 4-5 which is given the same formatting as Table 4-3. CNRR was able to solve 98% of the disruptions when nurses could only trade shifts from the second half of a scheduling period. The algorithm solved 100% of the disruptions that have feasible solutions. Only one disruption was impossible to solve and that disruption fit into Condition One.

In the third experimental set the majority of the disruptions were solved in the Impact Minimization phase. Unlike the other two experimental sets, this set required more LRB iterations. 49% of the disruptions required CNRR to progress to the Impact Minimization phase. Of those, 17% had a negligible impact and 22% had a small impact. Most (61%) required at least 100 LRB iterations with 17% requiring more than 400 iterations. Only one disruption in each of the first two sets required more than 400 LRB iterations.

Table 4-5: This table presents the ratio of the disruptions solved over 30 random experiments where each experiment had one or two disruptions. Disruptions varied in size from one day to three consecutive days. There were a total of 47 disruptions and 46 of those were solvable. Of the 47 disruptions, 53% were a single day, 38% were two days and 9% were three days. The first two rows present the ratio solved for the entire algorithm while the next six rows show the ratio solved by each algorithm stage. Rows that indicate "possible" are the ratios that consider those disruptions that do have feasible solutions.

	1 Day	2 Day	3 Day	Total
Ratio Solved	1.00	1.00	0.75	0.98
Ratio Possible Solved	1.00	1.00	1.00	1.00
Ratio SI	0.12	0.00	0.00	0.06
Ratio Possible SI	0.12	0.00	0.00	0.07
Ratio II	0.60	0.28	0.00	0.43
Ratio Possible II	0.60	0.28	0.00	0.43
Ratio IM	0.28	0.72	0.75	0.49
Ratio Possible IM	0.28	0.72	1.00	0.50

The runtime characteristics of CNR are most affected by the number of nurses being scheduled. This effect is $O(n^2)$. While adding nurses to CNR slows the algorithm, it also increases the solution space that can be explored and therefore produces better results with respect to preferences. Unlike CNR, CNR-ILS searches each nurse's schedule individually. As a result, the ILS runtime is primarily a product of schedule length. Longer scheduling periods allow the ILS to explore more potential shift trades which results in better results. The ILS in CNR-ILS is $O(n*d^2)$ where 'd' represents days. We include the 'n' in this equation to differentiate from the ILS runtime in the II stage of CNRR.

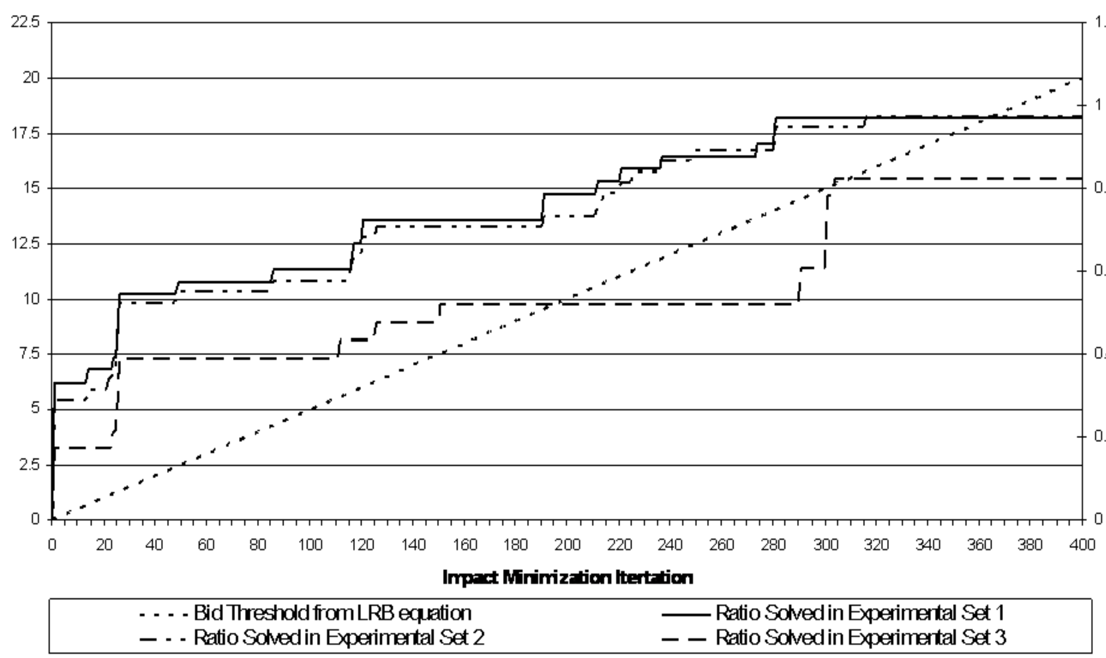

Figure 4-8: This chart shows how many iterations of the Impact Minimization (IM) phase were required for CNRR to solve a disruption. Runs that terminated prior to the IM phase or failed to find a solution are not included. When rerostering is confined to part of a scheduling period, as in experimental set three, and not the whole period, as in

experimental sets one and two, there will be larger impacts on nurse preferences. When CNRR can trade days that are part of a RO there is no improvement to the IM phase's effect on nurse preferences in which case the algorithm's only improvement is that it can solve more disruptions.

When rerostering, CNRR's runtime is affected by those of both CNR and CNR-ILS. Unlike CNR, the SI and IM stages of CNRR look to trade only those shifts that are disrupted. This difference causes the auction runtime to be on the order of O(n) with respect to nurses rather than $O(n^2)$. This improvement allows CNRR to use the Advanced Trading mechanism without serious impact to observed runtimes. The II phase needs to run the ILS only on the disrupted nurse's schedule. This difference means that the ILS in CNRR is on the order of $O(d^2)$ rather than $O(n*d^2)$. This difference is marginal because the largest effect is from the number of days not the number of nurses.

The most time intensive part of the CNRR algorithm is the Impact Minimization phase. The CNR auction is more complicated than the ILS and requires more runtime. Furthermore the Impact Minimization (IM) phase uses LRB while the Schedule Improvement (SI) phase does not. The use of LRB means that the IM phase can have multiple iterations, each having a new bid threshold. The SI phase has one iteration that uses a constant bid threshold of 0.0. The potential for multiple iterations in the IM phase means that the runtime of IM will be at least as large as the SI phase.

Figure 4-8 charts both the bid threshold that results from the LRB equation and the ratio of the cumulative number of disruptions that were solved when CNRR reached each iteration. The figure shows the algorithm requires more LRB iterations when there is

a decrease in the number of days considered (Experimental Set 3). The algorithm manipulates the entire schedule even though only the second half is being considered. Because the CNRR algorithm always manipulates the entire schedule, more LRB iterations require more time to complete.

4.4 DISCUSSION

CNRR is a modified version of the CNR-ILS rostering algorithm and represents the only current nurse rostering and rerostering system. CNRR minimizes the preference utility impact on the rostered nurses while adhering to Moz and Vaz Pato's definition of optimal – minimal difference from the reroster and the initial roster. This is accomplished this by assuring one-to-one shift trades and determining which shift trades to make via preference analysis.

While CNRR solved over 90% of the disruptions in all three of our experimental sets, it does have some shortcomings. The first is the potential for infeasibility. Since CNRR only allows one-to-one shift trades it cannot solve every possible disruption. In a three day disruption CNRR will only explore those solutions that have six different shift assignments (the three involved in the disruption and the three traded for the disruption). This limitation does not allow CNRR to explore the entire solution set.

A related shortcoming is that CNRR has no means to explore the impact of more complex trades on nurse preferences. While CNRR does limit the negative impact of disruptions it must do so within the context of one-to-one shift trade ratios. It is possible that other rerostering solutions exist that have smaller preference impacts but do not adhere to Moz and Vaz Pato's optimality definition.

Another limitation is that using LRB during the Impact Minimization phase requires an implied comparison of nurse utilities. During the development of CNR and CNR-ILS the direct comparison of the utility values from two different nurses was avoided. To accurately make these comparisons the utility functions of each nurse would need to be equally scaled. To avoid this complication, CNR and CNR-ILS ensured that decisions to alter a nurse's schedule were made by considering only that nurse's utility function. LRB allows negative preference impacts based on the numerical value of the impact on a nurse's utility. For these impacts to be consistent between the different nurses the utility functions must be of the same scale, something CNRR does not guarantee.

While CNRR is not perfect, it does present a promising rerostering methodology. It has the potential to be coupled with CNR-ILS to form a real-time scheduling and schedule recovery system. This system would have a high success rate in developing good rosters and solving disruptions to those rosters.

CHAPTER 5 CONCLUDING DISCUSSION AND FUTURE RESEARCH

Our vision for the nurse scheduling algorithms presented in this dissertation is a real-time scheduling paradigm where nurses are responsible for keeping their preferences up to date in the system and rosters are developed and reconstructed as needed. This real-time paradigm helps to minimize the need for designated scheduling staff.

CNRR, the culminating algorithm in this dissertation represents a unique and capable nurse scheduling method. The algorithm successfully divides the nurse rostering problem into a cost minimization problem and a preference maximization problem, models nurse preferences individually, presents a new agent based solution to the rostering and rerostering problem, and represents the first model designed to solve both the rostering and rerostering problems.

The Competitve Nurse Rostering Algorithms (CNRAs) were tested based on the staffing rules at Mike O'Callaghan Federal Hospital Air Force Medical Surgical Unit. This ward, being a military ward, differs from the general nursing industry. For example, the nurses working on this ward are not paid by the hour but rather by the annum. As a result, the cost minimization problem in these experiments required that each nurse work at least some minimum number of hours per scheduling period. By contrast, civilian nurses are often paid by the hour, may have differing pay scales, and may not be required to work a minimum number of hours per scheduling period. This ward also tends to use few pool nurses while some hospitals rely on float or pool nurses to satisfy varying demand. Because this is a military ward the nursing staff is subject to longer periods of absences due to required training and missions away from the hospital. Given these

differences, our paradigm has shown promise but requires more research to prove it applicability in the general nursing industry. While these are a few examples of where the test ward may be different it is not all inclusive. Overcoming these differences and extending our CNRA paradigm to the general nursing industry requires further research.

Nurses with varying pay scales can be accommodated in the CNRA paradigm by adding a single cost function to regulate shift trades between nurses. When two nurses that are paid at different rates or are paid different rate depending on what hour of the day they are working trade shifts, this function could measure the impact on the roster's cost. The overall cost could be controlled by setting an upper bound.

The use of extra staff could be regulated within the cost function through their naturally higher salaries or by imposing a subjective penalty for requiring the use of extra staffing beyond the core nursing staff.

Beyond simply generalizing the CNRA paradigm, it can be extended by researching modifications to the underlying methodologies and algorithm inputs. Some examples are as follows:

1. Using a nurse sign up model rather than an auction.
2. Trading shifts between more than two nurses at a time.
3. Integrating the ILS mechanism into the auction mechanism so that when there is staffing slack shift trades do not need to be made on a one-to-one ratio.
4. Altering the preference characteristics that are considered.
5. Developing an initial cost model rather than the simple input rosters we used.

6. Studying the affect of input rosters with different characteristics with respect to algorithm settings.

While the algorithm requires further research, it is promising and in our experiments proved to be a viable method for automatically solving both the rostering and rerostering problem.

DISCLAIMER

The views expressed in this article are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S. Government.

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APPENDIX A: A CHRONOLOGICAL TABLE OF STAFF SCHEDULING LITERATURE

Table A.1: This table defines seven common extensions to the traditional days-off, shift and tour scheduling. Specifically these definitions are used to categorize the papers listed in Table

Category	Definition
Preferences	Preference considerations of individual staff members are explicitly handled.
Rerostering	Fixes disruptions in an existing roster due to staff absenteeism.
Variable Shift Starting Times	Uses shifts that may have differing starting times.
Work-Rest Scheduling	Included the scheduling of break periods during scheduled work shifts.
Extra Staffing	Deals with the allocation and scheduling of overtime, or agency temporary staff. This does not include part time employees.
Scheduling Systems	These papers focus on the development of a useable scheduling system. These papers often outline a software system though it may just be a conceptual model.
Special Schedules	Research in this category utilizes nontraditional schedules. These include cyclical schedules and compressed schedules.

Table A.2: This table lists some of the literature related to staff scheduling. In this table the papers are sorted by publication year and categorized by what type of scheduling problem is addressed. Key papers, the rerostering column, and the preferences column are highlighted. These columns are singled out to focus on the part of the scheduling

problem addressed in our research. The three phases of the staff scheduling problem is shaded in grey: demand forecasting, demand covering, and rostering. Following the three phases are seven common research extensions: staff schedule preferences, rerostering, variable shift starting times, work-rest scheduling, extra staffing, scheduling systems, and special schedules. The seven extensions are defined in Table A.1.

Authors	Year	Survey Paper	Demand Forecast	Demand Covering	Rostering	Preferences	Rerostering	Variable Starting Times	Rest Breaks	Extra Staffing	Scheduling System	Special Schedules
Warner, D. Michael and Prawda, Juan	1972			X								
Abernathy, William et. al.	1973		X	X								
Segal, M	1974			X					X			
Miller, Holmes et. al.	1976			X	X	X						
Trived, Vandankumar and Warner, D. Michael	1976		X	X						X		
Warner, D. Michael	1976			X	X	X						
Gentzler, G.L. et. al.	1977								X			
Bechtold, Stephen	1979								X			
Morris, James and Showalter, Michael	1983			X								
Bechtold, Stephen et. al.	1984								X			
Bailey, James	1985			X				X				
Glover, Fred and McMillan, Claude	1986			X	X							
Rosenbloom, E.S. and Goertzen, N.F.	1987			X								X
Baxter, John and Mosby, Mark	1988			X	X	X						
Bechtold, Stephen	1988			X						X		
Bechtold, Stephen and Sumners, DeWitt	1988								X			
Okada, Mihoko and Okada, Masahiko	1988			X	X	X						
Okada, Mihoko and Okada, Masahiko	1988			X	X	X						
Ozkarahan, Irem and Bailey, James	1988			X							X	

Authors	Year	Survey Paper	Demand Forecast	Demand Covering	Rostering	Preferences	Rerostering	Variable Starting Times	Rest Breaks	Extra Staffing	Scheduling System	Special Schedules
Ozkarahan, Irem	1989			X	X	X					X	
Bechtold, Stephen and Jacobs, Larry	1990			X					X			
Bechtold, Stephen	1991								X			
Bechtold, Stephen et. al.	1991	X										
Easton, Fred and Rossin, Donald	1991			X								
Easton, Fred and Rossin, Donald	1991			X								
Hung, Rudy	1991			X	X							X
Kostreva, M.M., et. al.	1991				X							
Loucks, John and Jacobs, Robert	1991			X	X							
Ozkarahan, Irem	1991			X	X	X					X	
Chen, Jen-Gwo and Yeung, Tony W.	1992			X	X	X					X	
Okada, Mihoko	1992			X	X	X					X	
Siferd, Sue	1992	X										
Thompson, Gary M	1992			X								
Bechtold, Stephen and Thompson Gary	1993								X			
Brusco, MJ and Showalter, MJ	1993			X								
Franz, Lori and Miller, Janis	1993			X	X	X						
Hung, Rudy	1993			X	X							X
Hung, Rudy and Emmons, Hamilton	1993			X	X							X
Randhawa, Sabah and Sitompul, Darwin	1993			X	X						X	
Thompson, Gary M	1993			X					X			
Bechtold, Stephen and Brusco, Michael	1994			X								
Bechtold, Stephen and Brusco, Michael	1994			X								
Hung, Rudy	1994			X	X							X
Hung, Rudy	1994			X	X							X
Hung, Rudy	1994			X	X							X
Lauer, L. et. al.	1994			X	X						X	

Authors	Year	Survey Paper	Demand Forecast	Demand Covering	Rostering	Preferences	Rerostering	Variable Starting Times	Rest Breaks	Extra Staffing	Scheduling System	Special Schedules
Siferd, Sue and Benton, W.C.	1994		X									
Bechtold, Stephen and Brusco, Michael	1995			X								
Brusco, Michael and Jacobs, Larry	1995			X								
Thompson, Gary M	1995			X				X	X	X		
Thompson, Gary M	1995			X								
Brusco, Michael and Johns, Tony	1996			X								
Easton, Fred and Rossin, Donald	1996			X								
Thompson, Gary M	1996			X	X							
Easton, Fred and Rossin, Donald	1997			X						X		
Brusco, Michael	1998			X								
Brusco, Michael and Jacobs, Larry	1998			X								
Brusco, Michael and Jacobs, Larry	1998			X				X				
Dowsland, Kathryn	1998			X	X	X						
Millar, Harvey H. and Kiragu, Mona	1998			X	X							
Burns, R. and Narasimhan, R.	1999		X	X								X
Easton, Fred and Mansour, Nashat	1999		X	X								
Aickelin, Uwe and Dowsland, Kathryn	2000			X	X	X						
Brusco, Michael and Jacobs, Larry	2000			X				X	X			
Dowsland, K.A. and Thompson, J.M.	2000			X	X	X						
Jan, et. al.	2000			X	X							
Spyropoulos, Constantine	2000	X										
Thengvall, Benjamin et. al.	2000					X	X					
Brusco, Michael and Jacobs, Larry	2001			X				X				

Authors	Year	Survey Paper	Demand Forecast	Demand Covering	Rostering	Preferences	Rerostering	Variable Starting Times	Rest Breaks	Extra Staffing	Scheduling System	Special Schedules
Ferland, Jacques et. al.	2001			X								
Dias, Tiago M. et. al.	2003			X	X	X						
Ikegami, Atsuko and Niwa, Akira	2003			X	X	X						
Kawanaka, Hiroharu et. al.	2003			X	X	X						
Moz, Margarida and Vaz Pato, Margarida	2003						X					
Rosenberger, Jay et. al.	2003						X					
Yu, Gang et. al.	2003						X					
Aickelin, Uwe and Dowsland, Kathryn	2004			X	X	X						
Aickelin, Uwe and White, Paul	2004			X	X	X						
Alfares, Hesham	2004	X										
Bard, Jonathan F.	2004			X								
Bard, Jonathan F.	2004			X					X			
Ernst, A.T. et. al.	2004	X										
Ernst, A.T. et. al.	2004	X										
Eveborn, P. and Ronnqvist, M.	2004			X	X	X					X	
Isken, Mark	2004			X	X			X				
Moz, Margarida and Vaz Pato, Margarida	2004						X					
Winstanley, Graham	2004			X	X	X						
Azaiez, M.N. and Al Sharif, S.S	2005			X	X	X						
Bard, Jonathan and Purnomo, Hadi	2005		X	X	X	X						
Bard, Jonathan and Purnomo, Hadi	2005			X	X	X				X		
Easton, Fred	2005						X					
Fung, Spencer	2005			X	X							
Hattori, Hiromitsu et. al.	2005			X	X	X	X					
Bard, Jonathan and Purnomo, Hadi	2006			X	X	X				X		
Beddoe, Gareth and Petrovic, Sanja	2006			X	X							
Belien, Jeroen and	2006			X	X	X						

Authors	Year	Survey Paper	Demand Forecast	Demand Covering	Rostering	Preferences	Rerostering	Variable Starting Times	Rest Breaks	Extra Staffing	Scheduling System	Special Schedules
Demeulemeester, Erik												
Bhadury, J. and Radovilsky, Z.	2006			X	X							
Bratu, Stephane and Barnhart, Cynthia	2006						X					
Hochbaum, Dorit and Levin, Asaf	2006			X								X
Qi, Xiangtong and Bard, Jonathan F.	2006			X								
Trinkoff, Alison et. al.	2006											
Wright, Daniel	2006			X	X	X				X		
Bard, Jonathan and Purnomo, Hadi	2007			X	X	X						X
Gutjahr, Walter J. and Rauner, Marion S.	2007			X	X	X				X		
Huisman, Dennis	2007						X					
Moz, Margarida and Vaz Pato, Margarida	2007						X					

APPENDIX B: NURSE STAFFING RULES FROM MIKE O'CALLAGHAN FEDERAL HOSPITAL'S AIR FORCE MEDICAL/SURGICAL UNIT

1) Nurses work 12 hr shifts from 0700h-1900h or from 1900h-0700h.
2) Nurses must work 14 shifts in a 28 day scheduling period.
3) Every two days of paid vacation counts as one work day. Therefore if a nurse has 4 days of paid vacation during a scheduling period she needs to work only 12 shifts. The number of paid vacation days is rounded up so that if a nurse has 3 days of paid vacation she still only needs to work 12 shifts.
4) A nurse may have no more than two days off prior to taking paid vacation.
5) A nurse may have no more than two days off following a period of paid vacation.
6) If a nurse has two days off prior to paid vacation she may have only one day off following the paid vacation.
7) If a nurse has two days off following a paid vacation she may have only one day off prior to paid vacation.
8) A nurse may work no more than 3 days in a row.
9) A nurse may have no more than 4 days off in a row without taking paid vacation.
10) A nurse cannot work back-to-back shifts. For example working a night shift on Monday followed by a day shift on Tuesday.
11) There must be at least one level three nurse on duty during each shift to serve as the charge nurse.
12) Each night shift must be worked by at least 3 nurses that are not in a training status.

13) Each weekday day shift must be worked by at least 4 nurses that are not in a training status.

14) Each weekend day shift must be worked by at least 3 nurses that are not in a training status.

15) Every nurse can request up to four days off during a scheduling period. These request offs (ROs) are considered a courtesy and though they are usually granted they are not guaranteed. This is a hard constraint because nurses cannot request more that four days off in any scheduling period without taking vacation time.

16) *Nurses rotate shift assignments. They are assigned to work the night shift for three months then change assignments to the day shift. Nurses who have been on day shifts the longest rotate to night shifts as needed.

These rules are considered hard constraints that cannot be broken. In addition to these rules the scheduling staff at MOFH-AFMSU also takes into account the following guidelines as soft constraints:

1) Schedules should avoid single days off or on-off-on patterns.
2) Schedules should balance the workload over the scheduling period.
3) ROs should be granted if possible.
4) At least two days off should be given when transitioning between night and day shifts.
5) Weekends should not be split if possible.

*In CNRR nurses may work the opposite shift for a short time period to satisfy a disruption or to ensure roster feasibility. This action is governed by a cost penalty (C_3) added to the utility functions.

APPENDIX C: NURSE PREFERENCE WORKSHEET

Name:____________________________ **Date:**_____________

Please fill in your preferences as requested. A higher value indicates a stronger preference for having the indicated item in your 28 day schedule.

	Preference Points	
Schedules with 4 consecutive days off		Please assign UP TO 40 points for this section.
Schedules with 3 consecutive days off		
Schedules with 2 consecutive days off		
Total for schedule characteristics		

	A weekday off	**A weekend day off**
Which do you prefer? Do no assign points to this selection simply put an X under your preference if you have one.		

	Preference Points	**Start Day**	**End Day**
Requests off (Cannot be more than 4 total days. Requests for multiple days off as a group are treated all-or-nothing). Please assign UP TO 60 points for this section			
Total for requests offs			

APPENDIX D: AUTHOR AND COAUTHOR INFORMATION

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'F' indicates first author. 'C' indicates co-author.

APPENDIX E: RAW DATA TABLES

Table E-1: This table presents the results of our designed experimental runs used to tune the CNR algorithm.

CNR DOE RESULTS																							
				Trades				Utility				R/Os				Singles				Run Time			
Demand	Single off penalty	Bid threshold	Balance Penalty	Avg	Std	Min	Max	Avg	Std	Min	Max	Avg	Std	Min	Max	Avg	Std	Min	Max	Avg	Std	Min	Max
4	0	10	0	130.00	45.21	66.55	236.10	43.85	6.30	31.77	54.06	0.94	0.04	0.88	1.00	2.74	0.57	2.20	4.75	54.45	7.82	45.00	73.00
3	10	10	0	61.93	36.21	29.20	182.30	37.92	9.43	7.57	51.57	0.94	0.05	0.85	1.00	0.63	0.68	0.25	3.40	41.15	7.46	28	65
4	0	0	5	6.84	0.82	5.40	9.10	41.41	6.65	30.94	52.81	0.87	0.08	0.68	0.98	2.81	0.32	2.10	3.50	11.30	1.56	9.00	16.00
3	10	0	5	10.08	0.88	8.50	11.80	35.27	5.59	22.61	46.46	0.91	0.06	0.80	1.00	0.67	0.20	0.20	1.00	14.95	1.76	12	18
3	0	10	5	112.27	31.03	75.00	170.90	42.42	5.81	32.36	53.33	0.93	0.05	0.83	1.00	2.76	0.31	1.90	3.15	52.05	6.03	43	64
4	10	0	0	9.97	0.81	8.80	11.80	36.44	6.41	25.48	47.99	0.91	0.08	0.73	1.00	0.64	0.18	0.20	1.00	14.5	1.4	12	17
4	10	10	5	45.71	15.00	30.40	94.10	40.94	6.29	30.12	51.27	0.96	0.04	0.90	1.00	0.38	0.14	0.10	0.65	36.65	3.2	30	43
3	0	0	0	7.17	0.56	5.90	8.00	43.25	5.50	33.27	53.18	0.88	0.08	0.72	1.00	2.39	0.25	2.00	2.80	11.8	1.01	10	14

Table E-2: This table presents the performance of the CNR algorithm during our random experiments with respect to utility.

		CNR Utility Values																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	59.40	41.80	56.30	75.80	18.90	86.30	82.50	75.30	48.90	-5.00	70.80	77.30	51.60	12.60	12.50	15.10	21.10	3.11	26.70	56.80	44.39
	2	21.40	81.30	76.00	63.80	69.80	72.50	13.50	12.90	43.50	70.40	89.30	60.00	51.40	52.20	65.10	84.10	38.50	14.00	1.50	18.30	49.98
	3	-3.80	70.00	85.60	55.80	14.10	77.50	71.00	84.50	52.00	77.30	68.50	53.30	77.40	65.90	78.70	8.00	48.30	69.70	72.70	38.30	58.24
	4	83.10	63.30	23.30	11.00	3.00	62.00	19.00	80.50	0.30	57.80	56.10	9.20	20.50	22.80	32.60	17.60	44.00	12.30	33.50	59.00	35.55
	5	73.00	68.80	62.70	86.30	17.90	44.30	60.20	75.20	19.60	0.70	44.00	46.00	10.50	5.00	-2.50	7.50	45.40	52.10	30.30	64.70	40.59
	6	76.50	51.60	70.70	71.10	66.00	29.50	40.40	78.70	48.00	31.50	17.10	-5.30	18.60	8.00	45.70	17.70	15.30	32.70	82.60	45.50	42.10
	7	2.00	87.20	53.60	23.30	67.00	53.00	86.00	23.90	18.20	24.00	8.00	8.30	33.60	59.70	56.00	67.30	75.40	43.90	60.50	33.30	44.21
	8	67.50	24.00	21.00	78.80	20.90	71.00	57.20	46.80	7.80	59.10	71.20	20.50	48.80	22.50	17.70	44.00	18.90	28.00	52.20	-0.50	38.87
	9	60.70	62.30	56.80	2.20	0.00	89.30	-5.40	62.00	79.00	4.30	77.50	43.30	11.50	21.30	36.70	81.90	5.50	79.50	70.10	52.10	44.53
	10	64.80	8.00	46.70	-2.30	68.90	36.90	43.50	85.50	9.40	15.00	67.10	35.30	57.30	14.30	30.50	57.10	14.50	31.70	-1.10	39.80	36.15
	11	7.50	-13.70	28.30	48.70	63.00	20.60	39.70	36.30	60.00	4.50	-9.50	72.70	59.50	13.00	16.90	71.61	22.10	9.30	51.50	77.50	33.98
	12	76.90	54.00	26.60	75.70	-3.00	18.60	18.50	81.30	86.30	11.80	23.90	32.70	73.20	25.80	15.30	25.30	13.00	60.00	8.80	0.16	36.24
	13	43.10	79.90	7.70	81.40	72.90	28.30	73.00	84.20	44.30	51.40	22.00	24.10	59.50	-16.30	37.80	58.60	34.50	42.00	2.00	39.60	43.50
	14	5.80	14.50	51.00	18.70	1.00	70.00	52.80	26.80	50.80	21.90	25.70	59.60	68.50	24.50	77.80	21.10	75.20	37.10	27.90	23.10	37.69
	15	78.30	-12.40	6.50	25.10	82.20	13.50	9.00	13.90	16.90	49.20	80.50	62.10	18.00	45.50	52.40	45.90	12.00	53.80	7.00	-0.80	32.93
	16	59.00	67.60	-10.50	31.80	11.30	17.40	54.80	23.30	30.50	52.80	69.80	46.70	33.40	68.50	90.10	24.30	13.50	64.30	16.90	19.40	39.25
	17	59.30	47.80	83.00	62.50	78.90	42.70	51.20	31.70	20.50	47.80	77.90	-11.20	65.80	62.30	72.70	32.60	7.80	81.40	24.60	-6.20	46.66
	18	10.50	68.00	-6.70	81.10	65.30	52.20	8.30	78.60	-2.60	77.50	80.70	28.80	57.70	69.50	62.30	17.50	-2.80	10.10	25.50	41.00	41.13
	19	40.70	10.70	77.40	19.20	70.20	73.90	38.10	7.80	10.70	22.90	0.30	10.30	33.70	52.40	9.80	24.60	81.80	78.10	24.20	69.50	37.82
	20	71.80	30.80	24.50	55.60	30.20	67.70	19.90	25.50	76.10	63.20	12.80	43.30	8.30	14.30	55.00	40.60	22.30	67.50	48.80	54.60	41.64
AVG		47.88	45.28	42.03	48.28	40.93	51.36	41.66	51.74	36.01	36.91	47.69	35.85	42.94	32.19	43.16	38.12	30.32	43.53	33.31	36.26	41.27
STDV		29.56	30.93	29.90	29.34	31.45	24.46	26.34	29.07	26.76	26.82	31.72	25.14	22.54	25.09	26.53	24.47	24.58	25.52	25.18	25.02	27.20

Table E-3: This table presents the performance of the CNR algorithm using the AT mechanism during our random experiments with respect to utility.

		CNR Utility Values When Advanced Trading is Used																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	60.70	64.40	52.70	68.50	15.90	79.40	72.20	78.70	46.00	17.60	65.00	79.80	42.40	12.60	29.60	82.30	19.80	12.70	26.70	72.20	49.96
	2	23.90	48.40	76.00	78.60	69.00	62.50	-2.00	2.90	45.50	70.70	81.40	60.00	53.40	57.40	72.80	84.00	36.00	14.80	13.50	17.00	48.29
	3	8.75	69.80	74.80	49.30	18.40	74.80	59.80	60.30	75.00	75.90	43.80	54.00	77.40	62.10	78.70	23.70	57.50	72.50	72.40	51.50	58.02
	4	83.00	53.30	23.70	64.00	24.30	61.50	30.50	70.10	19.30	66.00	55.50	27.50	19.30	19.40	60.40	19.60	65.30	30.70	21.50	72.50	44.37
	5	85.50	83.80	77.70	63.90	22.30	16.00	43.00	74.60	29.30	55.00	59.00	63.30	11.80	16.30	-3.80	13.60	46.70	52.10	24.90	62.20	44.86
	6	74.00	57.30	58.20	73.00	66.00	22.60	40.70	76.10	34.00	29.00	15.80	11.70	4.80	2.30	44.50	19.70	1.10	18.20	9.10	45.50	35.18
	7	15.30	85.90	67.90	41.90	67.10	53.00	81.30	7.50	18.20	20.10	20.50	13.20	30.90	55.70	46.00	59.80	74.90	43.30	56.50	22.40	44.07
	8	66.30	24.00	16.50	76.30	19.70	50.90	73.40	68.00	25.00	37.80	73.00	15.50	67.80	17.30	15.20	44.00	13.70	29.50	53.00	18.00	40.25
	9	48.60	62.30	57.70	-9.80	13.60	83.40	10.80	75.40	77.70	-1.50	67.90	0.40	10.30	27.30	39.70	91.50	17.00	79.50	70.10	18.80	42.04
	10	64.20	6.00	45.60	9.40	69.70	44.70	38.50	68.40	-1.40	12.70	70.20	30.50	58.50	12.60	71.00	57.10	8.80	29.40	8.60	43.80	37.42
	11	8.80	68.10	26.00	38.70	47.50	20.20	45.40	37.50	73.80	15.80	20.80	76.00	67.30	14.70	47.50	76.70	19.40	42.20	77.30	76.90	45.03
	12	78.20	53.90	10.60	86.60	13.30	20.00	18.50	82.50	85.30	12.00	20.70	47.00	74.40	13.50	15.30	16.30	13.30	47.00	17.00	19.50	37.25
	13	28.60	66.70	25.00	78.90	58.50	4.10	59.50	81.70	42.00	71.80	35.80	53.60	69.00	8.50	10.30	58.60	24.50	-1.50	68.00	36.80	44.02
	14	11.30	18.20	39.80	-2.00	14.30	70.00	61.00	34.30	52.50	32.30	26.10	61.70	74.50	17.10	74.60	23.00	74.10	25.70	44.00	21.80	38.72
	15	74.50	16.50	16.40	25.10	82.20	7.30	13.00	24.40	16.90	47.90	83.90	55.30	12.00	45.50	60.70	55.80	13.50	52.50	12.70	10.50	36.33
	16	25.00	67.20	13.10	43.10	11.50	19.00	79.30	22.30	60.80	54.00	82.90	59.40	16.20	59.30	88.90	41.80	24.30	60.10	20.10	21.00	43.47
	17	67.10	46.50	83.00	52.50	76.50	38.10	50.00	30.60	23.00	46.60	77.90	25.30	59.60	68.50	80.90	14.60	11.90	83.00	65.20	22.00	51.14
	18	0.80	66.00	20.50	82.20	75.80	52.80	-3.20	77.70	14.90	68.30	78.20	28.80	67.10	71.50	62.30	13.60	14.90	17.00	16.50	21.10	42.34
	19	41.40	12.80	78.60	24.10	71.20	71.90	36.90	9.70	24.30	24.00	13.60	8.30	32.90	45.80	-0.30	5.10	74.50	76.80	32.70	69.50	37.69
	20	71.00	31.30	34.00	49.50	25.40	66.50	21.40	26.80	69.50	70.80	2.90	41.90	-0.70	14.00	26.30	28.70	22.60	72.50	50.50	61.50	39.32
AVG		46.85	50.12	44.89	49.69	43.11	45.94	41.50	50.48	41.58	41.34	49.75	40.66	42.45	32.07	46.03	41.48	31.69	42.90	38.02	39.23	42.99
STDV		28.84	23.97	25.34	28.51	26.97	25.63	25.99	28.23	25.14	24.43	27.62	23.63	27.28	23.10	28.53	27.44	24.45	25.24	23.97	22.82	25.83

Table E-4: This table presents the performance of the CNR algorithm during our random experiments with respect to the ratio of ROs granted.

		CNR Ratio of Request Offs Granted																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.50	0.95
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	0.50	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.90
	5	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	1.00	1.00	1.00	0.50	0.96
	11	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.88
	12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	13	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.50	1.00	0.67	1.00	1.00	1.00	1.00	0.50	1.00	0.50	1.00	0.88
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.93
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	0.98
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AVG		1.00	0.95	0.98	0.93	1.00	0.98	1.00	1.00	0.98	0.90	1.00	0.98	1.00	1.00	0.93	0.98	0.98	0.90	0.90	0.95	0.97
STDV		0.00	0.15	0.11	0.24	0.00	0.11	0.00	0.00	0.11	0.26	0.00	0.07	0.00	0.00	0.23	0.11	0.11	0.31	0.26	0.15	0.11

Table E-5: This table presents the performance of the CNR algorithm using the AT mechanism during our random experiments with respect to the ratio of ROs granted.

		CNR Ratio of Request Offs Granted When Advanced Trading is Used																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	0.50	1.00	1.00	0.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.50	1.00	0.93
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.85
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.50	0.93
	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	12	1.00	1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	13	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.50	0.00	1.00	1.00	0.85
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.93
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.95
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.95
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.95
	20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.93
AVG		0.98	0.98	0.93	0.95	1.00	0.93	1.00	0.98	1.00	0.93	1.00	0.95	0.90	1.00	0.93	0.90	0.98	0.90	0.93	0.88	0.95
STDV		0.11	0.11	0.24	0.22	0.00	0.24	0.00	0.11	0.00	0.24	0.00	0.22	0.31	0.00	0.24	0.31	0.11	0.31	0.24	0.32	0.17

Table E-6: This table presents the performance of the CNR algorithm during our random experiments with respect to the number of single days off.

		CNR Number of Single Days Off																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.25
	2	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.25
	3	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	1.00	0.40
	4	0.00	0.00	0.00	0.00	2.00	1.00	0.00	0.00	1.00	1.00	0.00	2.00	0.00	1.00	1.00	0.00	2.00	0.00	0.00	1.00	0.60
	5	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	1.00	2.00	0.00	1.00	1.00	0.00	1.00	0.00	0.00	0.00	0.45
	6	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20
	7	1.00	0.00	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25
	8	0.00	0.00	0.00	0.00	0.00	0.00	1.00	2.00	1.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.40
	9	0.00	0.00	1.00	0.00	1.00	0.00	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.35
	10	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.25
	11	0.00	3.00	0.00	0.00	0.00	0.00	2.00	0.00	1.00	1.00	2.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.60
	12	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.30
	13	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	2.00	0.00	0.00	0.00	2.00	1.00	0.00	0.50
	14	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.20
	15	0.00	3.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	1.00	0.45
	16	0.00	0.00	3.00	1.00	0.00	0.00	1.00	0.00	2.00	0.00	1.00	2.00	0.00	0.00	0.00	2.00	1.00	1.00	1.00	0.00	0.75
	17	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	1.00	1.00	0.00	0.00	0.00	1.00	1.00	0.35
	18	0.00	0.00	2.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.30
	19	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.25
	20	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	2.00	0.00	1.00	1.00	0.00	0.00	0.00	2.00	0.45
AVG		0.20	0.50	0.65	0.35	0.30	0.10	0.30	0.30	0.45	0.35	0.40	0.65	0.40	0.35	0.40	0.20	0.35	0.30	0.60	0.40	0.38
STDV		0.52	1.00	0.88	0.59	0.57	0.31	0.57	0.66	0.69	0.59	0.60	0.88	0.68	0.59	0.60	0.52	0.59	0.57	0.75	0.60	0.64

Table E-7: This table presents the performance of the CNR algorithm using the AT mechanism during our random experiments with respect to the number of single days off.

		CNR Number of Single Days Off When Advanced Trading is Used																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.15
	2	0.00	0.00	0.00	0.00	0.00	1.00	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20
	3	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20
	4	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.20
	5	0.00	0.00	0.00	2.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.30
	6	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	1.00	0.00	2.00	0.00	0.40
	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.15
	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.10
	9	1.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	11	0.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.05
	13	1.00	1.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	2.00	0.00	0.00	0.40
	14	0.00	0.00	1.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.20
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.15
	16	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.30
	17	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.15
	18	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.15
	19	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.20
	20	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	1.00	2.00	0.00	0.00	0.00	1.00	0.35
AVG		0.15	0.20	0.35	0.30	0.05	0.10	0.45	0.15	0.20	0.10	0.10	0.20	0.30	0.20	0.25	0.20	0.15	0.30	0.25	0.10	0.21
STDV		0.37	0.41	0.59	0.57	0.22	0.31	0.60	0.49	0.41	0.31	0.45	0.41	0.47	0.41	0.44	0.52	0.37	0.57	0.55	0.31	0.44

Table E-8: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the nurse utility values.

		CNR-ILS Utility Values																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	60.70	68.20	58.80	80.20	18.90	86.30	83.80	77.40	48.90	22.80	70.80	78.60	54.10	12.60	19.10	79.80	21.10	26.00	26.70	81.00	53.79
	2	25.10	81.30	76.00	78.60	72.60	75.00	19.00	26.00	46.00	71.40	89.30	62.00	60.30	66.90	77.10	84.10	38.50	17.50	13.30	30.30	55.52
	3	14.80	70.00	85.60	59.70	18.50	80.00	76.80	87.00	72.00	77.30	69.80	53.30	77.40	65.90	78.70	19.90	57.50	84.70	75.00	54.60	63.93
	4	86.00	63.30	62.40	66.00	26.30	76.30	19.00	80.50	18.00	69.00	57.00	9.20	20.50	30.70	57.90	24.50	69.00	12.30	33.50	71.30	47.64
	5	82.00	84.50	80.00	86.30	21.20	44.30	62.40	74.60	28.00	47.20	60.10	67.80	13.50	21.30	12.50	22.20	55.40	53.60	30.30	64.70	50.60
	6	76.50	68.90	70.70	71.10	67.20	29.50	40.40	78.70	48.00	31.50	21.40	16.80	22.40	15.80	45.70	17.70	15.30	32.70	86.20	45.50	45.10
	7	8.00	87.20	64.90	44.80	67.00	55.30	86.00	23.90	19.40	24.00	18.00	18.80	33.60	61.00	56.00	67.30	77.90	45.10	64.00	33.30	47.78
	8	67.50	27.00	21.00	87.00	23.00	72.70	68.30	73.50	23.40	59.10	76.00	20.50	69.20	22.50	21.10	45.30	24.40	44.00	52.20	19.00	45.84
	9	60.70	68.90	67.30	2.20	16.20	89.30	15.20	75.00	79.00	8.50	77.50	47.70	14.90	28.50	46.50	94.50	19.50	79.50	70.10	52.10	50.66
	10	65.40	8.00	46.70	9.20	68.90	48.10	42.30	89.70	9.40	15.00	71.20	40.00	66.80	15.60	71.00	57.10	24.00	31.70	13.60	50.30	42.20
	11	14.80	69.30	31.30	51.70	63.00	20.60	59.70	38.80	71.30	19.30	12.30	72.70	79.60	15.50	50.00	71.61	22.10	17.30	71.50	77.50	46.50
	12	87.60	54.00	42.30	87.90	13.30	21.30	18.50	85.30	86.30	15.80	25.10	48.90	75.70	27.00	16.50	31.60	13.00	60.00	17.00	18.20	42.27
	13	44.40	79.90	28.80	81.40	72.90	77.20	77.00	84.20	47.00	64.10	32.00	33.00	72.00	18.30	38.90	58.60	34.50	60.10	13.50	39.60	52.87
	14	10.10	18.50	51.00	19.70	17.90	70.00	73.80	28.00	51.00	21.90	25.70	59.60	73.30	24.50	79.00	21.80	76.50	37.10	51.20	24.70	41.77
	15	85.90	14.00	6.50	25.10	82.20	18.00	12.30	25.90	18.10	49.20	82.70	74.90	18.00	45.50	73.60	45.90	14.50	55.00	7.00	12.00	38.32
	16	59.00	67.60	30.60	43.00	13.30	17.40	79.30	28.50	70.10	57.00	84.60	52.80	37.60	69.80	90.10	35.30	28.60	67.60	25.70	19.40	48.87
	17	67.60	47.80	83.00	66.00	79.00	42.70	51.20	31.70	26.60	47.80	77.90	18.90	70.00	72.50	72.70	35.10	12.20	82.60	70.90	22.00	53.91
	18	18.00	68.00	16.60	83.70	69.30	54.60	8.30	78.60	12.30	77.50	80.70	28.80	57.70	73.50	64.30	16.30	12.20	27.80	28.00	41.00	45.86
	19	41.90	26.90	77.40	27.80	71.40	75.20	38.10	15.60	11.90	22.90	18.20	10.30	35.00	54.10	12.80	24.60	83.00	78.10	34.30	69.50	41.45
	20	73.00	34.80	44.50	57.70	30.20	69.40	19.90	28.00	76.10	72.00	21.60	44.60	28.30	15.50	70.00	49.71	23.10	73.80	48.80	82.00	48.15
AVG		52.45	55.41	52.27	56.46	45.62	56.16	47.57	56.55	43.14	43.67	53.60	42.96	49.00	37.85	52.68	45.15	36.12	49.33	41.64	45.40	48.15
STDV		27.98	24.98	23.94	27.45	26.96	24.41	27.40	27.65	25.66	23.72	27.86	22.43	23.91	22.86	25.06	24.29	24.37	23.52	24.61	23.00	25.29

Table E-9: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the nurse utility values when the demand for nurses is reduced from four to three per shift.

		CNR-ILS Utility Values When Nursing Demand is 3																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	60.70	70.80	58.80	80.20	18.90	86.30	83.80	77.40	48.90	20.20	70.80	78.60	54.10	12.60	19.10	79.80	21.10	26.00	26.70	81.00	53.79
	2	25.10	81.30	76.00	78.60	72.60	75.00	19.00	24.30	49.30	71.40	89.30	62.00	60.30	66.90	77.10	84.10	38.50	17.50	21.50	30.30	56.01
	3	14.80	70.00	85.60	60.90	18.50	80.00	76.80	87.00	81.30	77.30	69.80	53.30	77.40	65.90	78.70	21.20	57.50	84.70	75.00	58.90	64.73
	4	88.60	63.30	72.70	66.00	29.80	76.30	19.00	80.50	17.30	69.00	57.00	9.20	20.50	30.70	58.80	24.50	69.00	12.30	33.50	72.80	48.54
	5	82.00	84.50	80.00	86.30	21.20	44.30	62.40	74.60	28.00	49.80	60.10	68.50	13.50	21.30	18.80	22.20	55.40	53.60	30.30	64.70	51.08
	6	76.50	77.50	70.70	71.10	67.20	29.50	40.40	78.70	48.00	31.50	21.40	16.80	22.40	17.00	45.70	23.00	15.30	32.70	86.20	45.50	45.86
	7	15.30	87.20	64.90	44.80	67.00	55.30	86.00	23.90	20.70	25.30	19.30	18.80	33.60	61.00	56.00	67.30	77.90	46.40	64.00	33.30	48.40
	8	67.50	27.00	21.00	87.00	23.00	73.00	68.30	73.50	23.40	59.10	76.00	20.50	69.20	22.50	21.10	45.30	24.40	44.50	52.20	21.50	46.00
	9	60.70	70.20	69.70	2.20	15.00	89.30	16.40	83.70	79.00	8.50	77.50	47.70	14.90	28.50	46.50	98.30	19.50	79.50	70.10	52.10	51.47
	10	65.40	8.00	46.70	9.20	68.90	48.10	42.30	89.70	9.40	15.00	71.50	40.00	66.80	15.60	71.00	57.10	28.50	31.70	22.90	50.30	42.91
	11	12.40	72.40	31.30	51.70	63.00	20.60	59.70	38.80	76.30	20.50	25.30	72.70	79.60	16.70	50.00	72.90	24.60	17.30	83.00	77.50	48.32
	12	87.60	54.00	42.30	89.10	20.90	21.30	18.50	85.30	86.30	19.50	25.10	50.20	75.70	29.50	16.50	31.60	13.00	60.00	18.50	19.40	43.22
	13	44.40	79.90	31.30	81.40	72.90	79.70	77.00	84.20	47.50	75.60	37.10	64.80	72.00	19.50	38.90	58.60	34.50	60.10	60.00	39.60	57.95
	14	12.20	18.50	51.00	22.60	18.80	70.00	80.50	34.30	51.00	21.90	25.70	59.60	74.50	24.50	79.00	21.80	77.70	37.10	51.20	27.60	42.98
	15	85.90	16.50	16.70	25.10	82.20	19.30	14.30	25.90	18.10	49.20	82.70	74.90	18.00	45.50	73.60	55.90	14.50	56.30	10.50	12.00	39.86
	16	59.00	67.60	37.90	44.30	15.50	17.40	80.50	28.50	70.50	59.50	83.30	66.70	40.60	69.80	90.10	35.30	28.60	67.60	25.70	19.40	50.39
	17	72.40	47.80	83.00	67.30	79.00	42.70	51.20	31.70	26.60	47.80	77.90	18.90	70.00	72.50	80.90	35.10	16.30	82.60	70.90	22.00	54.83
	18	19.30	68.00	24.30	83.70	69.30	54.60	8.30	78.60	12.30	82.50	79.50	28.80	57.70	73.50	64.30	16.30	14.70	27.80	28.00	41.00	46.63
	19	41.90	26.90	77.40	32.20	71.40	75.20	38.10	16.80	26.60	22.90	18.20	10.30	35.00	54.10	12.80	24.60	83.00	78.10	34.30	69.50	42.47
	20	74.30	34.80	44.50	60.90	30.20	72.40	19.90	28.00	76.10	72.00	21.60	44.60	31.70	19.30	71.30	49.70	23.10	73.40	48.80	82.00	48.93
AVG		53.30	56.31	54.29	57.23	46.27	56.52	48.12	57.27	44.83	44.93	54.46	45.35	49.38	38.35	53.51	46.23	36.86	49.46	45.67	46.02	49.22
STDV		27.71	25.29	22.37	27.11	26.29	24.53	27.64	27.72	25.87	24.64	26.57	23.33	23.77	22.44	25.04	24.54	23.98	23.50	23.41	22.84	25.08

Table E-10: This table presents the performance of the CNR-ILS algorithm using the AT mechanism during our random experiments with respect to the nurse utility values.

		CNR-ILS Utility Values When Advanced Trading is Used																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	60.70	69.40	58.80	74.80	15.90	80.70	84.00	78.70	47.60	18.90	70.60	79.80	52.30	12.60	29.60	82.30	21.10	24.30	26.70	79.80	53.43
	2	25.10	81.40	76.00	78.60	70.60	71.00	19.00	26.00	46.80	70.70	89.30	62.00	53.40	60.30	77.10	85.30	37.60	16.30	20.30	27.80	54.73
	3	13.60	72.20	85.60	58.30	18.50	79.80	82.50	67.00	86.30	76.00	64.00	54.30	77.40	65.90	78.70	24.50	62.00	83.40	75.00	58.90	64.20
	4	85.60	66.70	72.70	66.00	26.30	75.00	30.50	80.50	20.50	72.00	56.10	27.50	27.50	19.40	62.00	24.50	70.30	31.60	36.10	75.00	51.29
	5	86.80	84.50	80.00	86.30	22.30	45.50	60.10	75.30	30.50	59.30	60.10	68.00	13.50	17.50	15.00	14.80	46.70	53.50	24.90	63.40	50.40
	6	76.50	68.90	60.70	73.00	66.00	29.50	41.70	78.70	40.00	30.30	18.90	12.90	16.20	21.80	45.70	23.00	23.00	32.70	80.30	45.50	44.27
	7	16.00	87.20	67.90	42.30	67.10	54.80	82.50	24.70	18.20	25.30	20.50	13.20	34.50	60.90	56.00	62.30	76.20	44.60	60.50	34.10	47.44
	8	67.70	27.00	21.00	80.00	20.90	56.80	73.40	72.30	26.30	50.30	73.70	20.50	70.10	18.60	17.70	45.30	13.70	41.30	55.60	18.00	43.51
	9	60.70	66.80	69.70	3.30	14.90	84.60	12.00	75.40	81.40	7.00	72.50	46.40	13.60	28.50	47.70	98.30	17.00	79.50	70.10	18.80	48.41
	10	65.40	8.00	46.90	9.70	69.70	48.10	42.30	70.30	9.40	12.70	71.50	35.30	59.50	24.50	71.00	57.10	17.00	31.90	11.10	70.00	41.57
	11	13.60	72.40	32.30	51.70	63.00	23.40	58.40	45.20	80.00	18.30	27.70	76.00	78.30	15.90	48.80	76.70	23.30	47.30	83.00	78.10	50.67
	12	83.20	53.90	11.90	87.90	18.90	22.30	19.80	86.50	86.30	13.30	22.60	52.70	75.70	27.00	15.30	20.20	14.50	58.50	17.00	25.40	40.65
	13	45.00	79.90	31.30	80.20	72.90	5.30	67.50	81.70	43.80	75.60	37.10	62.50	70.80	17.00	13.80	58.60	31.00	68.90	70.00	37.10	52.50
	14	12.20	18.30	51.00	19.90	24.60	70.00	79.30	37.50	52.50	34.50	27.40	61.70	74.50	22.00	80.80	23.00	75.30	37.10	47.80	26.30	43.79
	15	81.40	17.80	16.70	25.10	83.10	16.00	14.30	24.40	20.60	47.90	83.90	70.60	18.00	45.50	60.70	58.60	14.40	55.00	18.30	10.50	39.14
	16	25.00	68.00	39.20	46.10	14.50	19.00	79.30	23.50	70.50	58.30	90.50	73.30	36.30	72.00	88.90	50.40	24.30	78.80	34.70	23.50	50.81
	17	68.30	46.50	83.00	67.30	79.00	39.40	50.00	30.60	24.20	47.80	77.90	25.30	71.30	68.50	82.70	23.40	17.60	83.40	66.40	22.00	53.73
	18	17.00	66.00	20.50	84.70	77.00	55.30	10.70	77.70	16.20	76.30	80.70	31.30	67.10	71.50	65.20	13.60	14.90	18.30	26.80	24.90	45.79
	19	41.40	12.80	78.60	24.10	71.20	71.90	36.90	9.70	24.30	24.00	13.60	8.30	32.90	45.80	-0.30	5.10	74.50	76.80	32.70	69.50	37.69
	20	71.00	31.30	34.00	49.50	25.40	66.50	21.40	26.80	69.50	70.80	2.90	41.90	-0.70	14.00	26.30	28.70	22.60	72.50	50.50	61.50	39.32
AVG		50.81	54.95	51.89	55.44	46.09	50.75	48.28	54.63	44.75	44.47	53.08	46.18	47.11	36.46	49.14	43.79	34.85	51.79	45.39	43.51	47.67
STDV		27.74	26.09	24.51	26.99	27.05	24.58	26.74	26.19	25.89	24.49	28.53	23.09	26.11	22.10	27.49	27.46	23.41	22.48	23.34	23.40	25.44

Table E-11: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the ratio of ROs granted.

		CNR-ILS Ratio of Request Offs Granted																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.95
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98
	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.95
	12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	0.67	1.00	1.00	1.00	1.00	0.50	1.00	0.50	1.00	0.91
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.95
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AVG		1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00	0.93	1.00	0.98	1.00	1.00	1.00	1.00	0.98	0.90	0.93	0.98	0.98
STDV		0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.07	0.00	0.00	0.00	0.00	0.11	0.31	0.24	0.11	0.07

Table E-12: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the ratio of ROs granted when the demand for nurses is decreased from four to three per shift.

		CNR-ILS Ratio of Request Offs Granted When Nursing Demand is 3																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.95
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	0.98
	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.95
	12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.98
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.95
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AVG		1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.90	0.95	0.98	0.99
STDV		0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.31	0.22	0.11	0.06

Table E-13: This table presents the performance of the CNR-ILS algorithm using the AT mechanism during our random experiments with respect to the ratio of ROs granted.

		CNR-ILS Ratio of Request Offs Granted When Advanced Trading is Used																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.90
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.95
	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	12	1.00	1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90
	13	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.50	1.00	1.00	1.00	0.90
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.95
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.95
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.95
	20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	0.93
AVG		0.98	1.00	0.95	0.95	1.00	0.98	1.00	1.00	1.00	0.93	1.00	1.00	0.95	1.00	0.93	0.90	0.98	1.00	0.95	0.90	0.97
STDV		0.11	0.00	0.22	0.22	0.00	0.11	0.00	0.00	0.00	0.24	0.00	0.00	0.22	0.00	0.24	0.31	0.11	0.00	0.22	0.31	0.12

Table E-14: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the number of single days off.

		CNR-ILS Number of Single Days Off																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	4	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.05
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.10
	17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.05
	18	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AVG		0.00	0.00	0.15	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.10	0.15	0.05	0.00	0.05	0.05	0.00	0.00	0.05	0.00	0.04
STDV		0.00	0.00	0.37	0.00	0.00	0.00	0.00	0.00	0.22	0.22	0.31	0.49	0.22	0.00	0.22	0.22	0.00	0.00	0.22	0.00	0.13

Table E-15: This table presents the performance of the CNR-ILS algorithm during our random experiments with respect to the number of single days off when the demand for nurses is reduced from four to three per shift.

		CNR-ILS Number of Single Days Off When Nursing Demand is 3																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.05
	14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.05
	17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AVG		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.10	0.05	0.00	0.00	0.05	0.00	0.00	0.05	0.00	0.02
STDV		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.22	0.45	0.22	0.00	0.00	0.22	0.00	0.00	0.22	0.00	0.08

Table E-16: This table presents the performance of the CNR-ILS algorithm using the AT mechanism during our random experiments with respect to the number of single days off.

		CNR-ILS Number of Single Days Off When Advanced Trading is Used																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.05
	6	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.05
	16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	19	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.20
	20	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	2.00	0.00	0.00	0.00	1.00	0.30
AVG		0.00	0.05	0.10	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.05	0.00	0.10	0.10	0.15	0.15	0.05	0.00	0.00	0.05	0.05
STDV		0.00	0.22	0.31	0.00	0.00	0.00	0.00	0.00	0.22	0.22	0.22	0.00	0.31	0.31	0.37	0.49	0.22	0.00	0.00	0.22	0.16

Table E-17: This table presents the performance of the IP model during our random experiments with respect to the nurse utility values.

		Integer Program Utility Values																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	54.80	61.80	52.70	73.80	18.80	74.40	83.10	77.40	42.10	7.50	71.80	78.40	51.00	9.50	19.10	76.90	13.93	19.36	24.50	79.57	49.52
	2	25.10	75.90	50.60	73.10	70.30	72.50	16.00	21.80	38.30	69.60	90.50	62.00	52.70	55.50	67.80	76.70	37.57	14.75	9.00	7.50	49.36
	3	13.00	68.60	83.00	53.00	16.30	74.00	71.00	82.10	63.30	70.70	69.50	53.30	74.90	62.60	78.70	11.80	44.50	72.50	72.43	54.58	59.49
	4	86.10	60.50	72.70	61.00	20.50	65.50	5.00	78.00	11.50	66.00	56.40	14.80	17.70	27.90	52.60	24.50	64.00	27.67	34.86	69.25	45.82
	5	77.00	71.70	67.70	74.50	16.60	40.00	55.00	73.40	13.30	55.00	59.00	66.80	6.30	17.50	12.50	12.30	46.36	53.32	18.04	56.86	44.66
	6	76.50	57.50	50.20	66.30	64.70	23.20	37.60	67.40	42.00	24.60	15.00	9.00	11.90	11.00	42.90	15.60	0.75	32.71	75.50	34.57	37.95
	7	11.30	79.60	64.90	40.40	65.90	51.80	71.50	20.00	15.40	20.10	16.00	14.20	33.40	59.40	44.00	58.30	73.17	42.79	56.25	33.68	43.60
	8	65.90	18.00	15.00	70.50	17.10	73.20	64.50	62.00	18.70	58.80	73.00	16.80	68.50	22.50	9.60	44.20	12.86	40.50	50.14	5.67	40.37
	9	56.40	63.50	65.00	13.50	9.90	83.90	12.90	64.10	79.50	18.90	72.90	43.30	9.80	18.50	46.70	83.50	11.00	75.71	69.00	53.36	47.57
	10	64.80	5.00	46.00	9.40	69.10	47.00	30.20	64.80	9.40	10.40	69.30	21.00	67.30	14.30	55.00	54.60	15.00	26.89	22.93	71.25	38.68
	11	9.90	69.30	27.30	48.70	47.00	14.60	54.60	42.50	73.80	7.30	29.40	61.30	72.00	10.10	45.70	72.86	19.25	40.67	65.75	78.14	44.51
	12	71.30	51.60	42.90	78.50	6.90	13.80	15.20	81.20	77.50	14.50	17.70	44.50	73.50	12.40	11.90	20.17	13.00	60.00	15.25	20.71	37.13
	13	36.80	71.70	20.00	78.30	73.60	76.30	59.00	83.80	44.00	68.50	30.80	61.30	67.50	7.00	38.20	55.96	59.00	67.64	63.00	36.43	54.94
	14	7.80	18.30	51.00	7.90	16.50	61.00	77.60	30.50	50.80	16.00	24.90	59.60	67.50	24.50	77.40	20.54	73.32	36.79	51.21	22.67	39.79
	15	74.50	7.20	12.60	25.10	82.20	5.00	7.80	24.40	18.10	49.20	78.00	65.30	11.50	41.80	67.70	55.92	12.29	52.25	10.50	10.50	35.59
	16	59.00	67.20	34.30	48.50	9.50	16.80	72.80	26.00	69.70	49.80	76.00	62.80	34.70	68.00	72.50	41.83	16.18	61.36	31.61	15.11	46.68
	17	68.30	47.80	74.10	56.30	72.90	38.10	49.40	29.80	20.50	39.70	74.10	17.60	65.00	64.00	82.20	31.36	12.00	70.64	66.43	5.50	49.29
	18	17.00	63.00	14.50	79.60	69.30	46.30	0.00	77.70	16.20	66.50	79.60	21.00	67.10	66.00	62.30	13.39	9.50	19.50	25.00	28.00	42.07
	19	41.30	24.10	78.60	16.80	65.50	67.00	36.30	9.00	24.40	17.80	10.30	10.30	24.60	52.40	6.80	24.57	66.17	75.46	31.17	65.25	37.39
	20	57.00	18.50	43.90	54.80	9.80	67.70	21.10	28.00	71.50	72.00	7.00	36.20	27.70	11.50	54.30	49.71	22.33	71.25	44.50	72.33	42.06
AVG		48.69	50.04	48.35	51.50	41.12	50.61	42.03	52.20	40.00	40.15	51.06	40.98	45.23	32.82	47.40	42.24	31.11	48.09	41.85	41.05	44.32
STDV		26.13	24.83	22.20	24.70	28.50	24.80	27.11	25.85	24.98	24.70	28.29	23.21	25.27	22.87	24.47	24.00	24.35	20.55	22.18	26.16	24.93

Table E-18: This table presents the performance of the IP model during our random experiments with respect to the ratio of ROs granted.

		Integer Program Ratio of Request Offs Granted																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	14	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.95
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20	1.00	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
AVG		1.00	0.98	0.98	0.95	1.00	0.98	1.00	1.00	0.95	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.95	1.00	0.99
STDV		0.00	0.11	0.11	0.22	0.00	0.11	0.00	0.00	0.15	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.06

Table E-19: This table presents the performance of the IP model during our random experiments with respect to the number of single days off.

		Integer Program Number of Single Days Off																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.05
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	13	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	20	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
AVG		0.05	0.05	0.00	0.00	0.00	0.00	0.10	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.02
STDV		0.22	0.22	0.00	0.00	0.00	0.00	0.31	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.06

Table E-20: This table presents the difference between the nurse utility values from CNR algorithm and the IP model during our random experiments. The difference is calculated as the CNR value minus the IP value.

		Nurse Utility Difference (CNR-IP)																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	4.60	-20.00	3.60	2.00	0.10	11.90	-0.60	-2.10	6.80	-12.50	-1.00	-1.10	0.60	3.10	-6.60	-61.80	7.17	-16.25	2.20	-22.77	-5.13
	2	-3.70	5.40	25.40	-9.30	-0.50	0.00	-2.50	-8.90	5.20	0.80	-1.20	-2.00	-1.30	-3.30	-2.70	7.40	0.93	-0.75	-7.50	10.80	0.61
	3	-16.80	1.40	2.60	2.80	-2.20	3.50	0.00	2.40	-11.30	6.60	-1.00	0.00	2.50	3.30	0.00	-3.80	3.80	-2.80	0.27	-16.28	-1.25
	4	-3.00	2.80	-49.40	-50.00	-17.50	-3.50	14.00	2.50	-11.20	-8.20	-0.30	-5.60	2.80	-5.10	-20.00	-6.90	-20.00	-15.37	-1.36	-10.25	-10.28
	5	-4.00	-2.90	-5.00	11.80	1.30	4.30	5.20	1.80	6.30	-54.30	-15.00	-20.80	4.20	-12.50	-15.00	-4.80	-0.96	-1.22	12.26	7.84	-4.07
	6	0.00	-5.90	20.50	4.80	1.30	6.30	2.80	11.30	6.00	6.90	2.10	-14.30	6.70	-3.00	2.80	2.10	14.55	-0.01	7.10	10.93	4.15
	7	-9.30	7.60	-11.30	-17.10	1.10	1.20	14.50	3.90	2.80	3.90	-8.00	-5.90	0.20	0.30	12.00	9.00	2.23	1.11	4.25	-0.38	0.61
	8	1.60	6.00	6.00	8.30	3.80	-2.20	-7.30	-15.20	-10.90	0.30	-1.80	3.70	-19.70	0.00	8.10	-0.20	6.04	-12.50	2.06	-6.17	-1.50
	9	4.30	-1.20	-8.20	-11.30	-9.90	5.40	-18.30	-2.10	-0.50	-14.60	4.60	0.00	1.70	2.80	-10.00	-1.60	-5.50	3.79	1.10	-1.26	-3.04
	10	0.00	3.00	0.70	-11.70	-0.20	-10.10	13.30	20.70	0.00	4.60	-2.20	14.30	-10.00	0.00	-24.50	2.50	-0.50	4.81	-24.03	-31.45	-2.54
	11	-2.40	-83.00	1.00	0.00	16.00	6.00	-14.90	-6.20	-13.80	-2.80	-38.90	11.40	-12.50	2.90	-28.80	-1.25	2.85	-31.37	-14.25	-0.64	-10.53
	12	5.60	2.40	-16.30	-2.80	-9.90	4.80	3.30	0.10	8.80	-2.70	6.20	-11.80	-0.30	13.40	3.40	5.13	0.00	0.00	-6.45	-20.55	-0.88
	13	6.30	8.20	-12.30	3.10	-0.70	-48.00	14.00	0.40	0.30	-17.10	-8.80	-37.20	-8.00	-23.30	-0.40	2.64	-24.50	-25.64	-61.00	3.17	-11.44
	14	-2.00	-3.80	0.00	10.80	-15.50	9.00	-24.80	-3.70	0.00	5.90	0.80	0.00	1.00	0.00	0.40	0.56	1.88	0.31	-23.31	0.43	-2.10
	15	3.80	-19.60	-6.10	0.00	0.00	8.50	1.20	-10.50	-1.20	0.00	2.50	-3.20	6.50	3.70	-15.30	-10.02	-0.29	1.55	-3.50	-11.30	-2.66
	16	0.00	0.40	-44.80	-16.70	1.80	0.60	-18.00	-2.70	-39.20	3.00	-6.20	-16.10	-1.30	0.50	17.60	-17.53	-2.68	2.94	-14.71	4.29	-7.44
	17	-9.00	0.00	8.90	6.20	6.00	4.60	1.80	1.90	0.00	8.10	3.80	-28.80	0.80	-1.70	-9.50	1.24	-4.20	10.76	-41.83	-11.70	-2.63
	18	-6.50	5.00	-21.20	1.50	-4.00	5.90	8.30	0.90	-18.80	11.00	1.10	7.80	-9.40	3.50	0.00	4.11	-12.30	-9.40	0.50	13.00	-0.95
	19	-0.60	-13.40	-1.20	2.40	4.70	6.90	1.80	-1.20	-13.70	5.10	-10.00	0.00	9.10	0.00	3.00	0.03	15.63	2.64	-6.97	4.25	0.42
	20	14.80	12.30	-19.40	0.80	20.40	0.00	-1.20	-2.50	4.60	-8.80	5.80	7.10	-19.40	2.80	0.70	-9.11	-0.03	-3.75	4.30	-17.73	-0.42
AVG		-0.82	-4.77	-6.33	-3.22	-0.19	0.76	-0.37	-0.46	-3.99	-3.24	-3.38	-5.13	-2.29	-0.63	-4.24	-4.12	-0.79	-4.56	-8.54	-4.79	-3.05
STDV		6.78	20.32	18.28	13.84	8.93	12.50	11.49	7.55	11.65	14.41	10.06	13.11	8.20	7.22	11.96	14.97	9.66	10.68	17.71	12.51	12.54
Paired T		-0.54	-1.05	-1.55	-1.04	-0.10	0.27	-0.14	-0.27	-1.53	-1.01	-1.50	-1.75	-1.25	-0.39	-1.59	-1.23	-0.37	-1.91	-2.16	-1.71	-4.87
LCI		-1.35	-6.37	-7.77	-4.32	-0.90	-0.23	-1.28	-1.06	-4.91	-4.38	-4.17	-6.16	-2.94	-1.20	-5.19	-5.30	-1.56	-5.40	-9.95	-5.78	-3.28
UCI		-0.28	-3.16	-4.88	-2.12	0.51	1.74	0.54	0.14	-3.07	-2.10	-2.58	-4.09	-1.64	-0.06	-3.29	-2.93	-0.03	-3.71	-7.14	-3.80	-2.83

Table E-21: This table presents the difference between the nurse utility values from CNR-ILS algorithm and the IP model during our random experiments. The difference is calculated as the CNR-ILS value minus the IP value.

		Nurse Utility Difference ((CNR-ILS)-IP)																				
		Run																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**AVG**
Nurse	**1**	5.90	6.40	6.10	6.40	0.10	11.90	0.70	0.00	6.80	15.30	-1.00	0.20	3.10	3.10	0.00	2.90	7.17	6.64	2.20	1.43	4.27
	2	0.00	5.40	25.40	5.50	2.30	2.50	3.00	4.20	7.70	1.80	-1.20	0.00	7.60	11.40	9.30	7.40	0.93	2.75	4.30	22.80	6.15
	3	1.80	1.40	2.60	6.70	2.20	6.00	5.80	4.90	8.70	6.60	0.30	0.00	2.50	3.30	0.00	8.10	13.00	12.20	2.57	0.02	4.43
	4	-0.10	2.80	-10.30	5.00	5.80	10.80	14.00	2.50	6.50	3.00	0.60	-5.60	2.80	2.80	5.30	0.00	5.00	-15.37	-1.36	2.05	1.81
	5	5.00	12.80	12.30	11.80	4.60	4.30	7.40	1.20	14.70	-7.80	1.10	1.00	7.20	3.80	0.00	9.90	9.04	0.28	12.26	7.84	5.94
	6	0.00	11.40	20.50	4.80	2.50	6.30	2.80	11.30	6.00	6.90	6.40	7.80	10.50	4.80	2.80	2.10	14.55	-0.01	10.70	10.93	7.15
	7	-3.30	7.60	0.00	4.40	1.10	3.50	14.50	3.90	4.00	3.90	2.00	4.60	0.20	1.60	12.00	9.00	4.73	2.31	7.75	-0.38	4.17
	8	1.60	9.00	6.00	16.50	5.90	-0.50	3.80	11.50	4.70	0.30	3.00	3.70	0.70	0.00	11.50	1.10	11.54	3.50	2.06	13.33	5.46
	9	4.30	5.40	2.30	-11.30	6.30	5.40	2.30	10.90	-0.50	-10.40	4.60	4.40	5.10	10.00	-0.20	11.00	8.50	3.79	1.10	-1.26	3.09
	10	0.60	3.00	0.70	-0.20	-0.20	1.10	12.10	24.90	0.00	4.60	1.90	19.00	-0.50	1.30	16.00	2.50	9.00	4.81	-9.33	-20.95	3.52
	11	4.90	0.00	4.00	3.00	16.00	6.00	5.10	-3.70	-2.50	12.00	-17.10	11.40	7.60	5.40	4.30	-1.25	2.85	-23.37	5.75	-0.64	1.99
	12	16.30	2.40	-0.60	9.40	6.40	7.50	3.30	4.10	8.80	1.30	7.40	4.40	2.20	14.60	4.60	11.43	0.00	0.00	1.75	-2.51	5.14
	13	7.60	8.20	8.80	3.10	-0.70	0.90	18.00	0.40	3.00	-4.40	1.20	-28.30	4.50	11.30	0.70	2.64	-24.50	-7.54	-49.50	3.17	-2.07
	14	2.30	0.20	0.00	11.80	1.40	9.00	-3.80	-2.50	0.20	5.90	0.80	0.00	5.80	0.00	1.60	1.26	3.18	0.31	-0.01	2.03	1.97
	15	11.40	6.80	-6.10	0.00	0.00	13.00	4.50	1.50	0.00	0.00	4.70	9.60	6.50	3.70	5.90	-10.02	2.21	2.75	-3.50	1.50	2.72
	16	0.00	0.40	-3.70	-5.50	3.80	0.60	6.50	2.50	0.40	7.20	8.60	-10.00	2.90	1.80	17.60	-6.53	12.42	6.24	-5.91	4.29	2.18
	17	-0.70	0.00	8.90	9.70	6.10	4.60	1.80	1.90	6.10	8.10	3.80	1.30	5.00	8.50	-9.50	3.74	0.20	11.96	4.47	16.50	4.62
	18	1.00	5.00	2.10	4.10	0.00	8.30	8.30	0.90	-3.90	11.00	1.10	7.80	-9.40	7.50	2.00	2.91	2.70	8.30	3.00	13.00	3.79
	19	0.60	2.80	-1.20	11.00	5.90	8.20	1.80	6.60	-12.50	5.10	7.90	0.00	10.40	1.70	6.00	0.03	16.83	2.64	3.13	4.25	4.06
	20	16.00	16.30	0.60	2.90	20.40	1.70	-1.20	0.00	4.60	0.00	14.60	8.40	0.60	4.00	15.70	0.00	0.77	2.55	4.30	9.67	6.09
AVG		3.76	5.37	3.92	4.96	4.50	5.56	5.39	4.35	3.14	3.52	2.54	1.99	3.77	5.03	5.28	2.91	5.01	1.24	-0.21	4.35	3.82
STDV		5.40	4.55	8.37	6.28	5.36	3.89	5.64	6.40	5.80	6.32	6.03	9.47	4.45	4.16	6.73	5.51	8.63	8.39	12.65	9.01	6.97
Paired T		3.12	5.28	2.09	3.53	3.75	6.38	4.28	3.04	2.42	2.49	1.88	0.94	3.79	5.41	3.51	2.36	2.59	0.66	-0.08	2.16	10.97
LCI		3.33	5.00	3.26	4.46	4.07	5.25	4.94	3.84	2.68	3.02	2.06	1.24	3.41	4.70	4.75	2.47	4.32	0.57	-1.21	3.64	3.70
UCI		4.19	5.73	4.58	5.45	4.92	5.86	5.84	4.86	3.60	4.02	3.01	2.73	4.12	5.36	5.81	3.35	5.69	1.90	0.79	5.07	3.95

Table E-22: This table presents the difference between the nurse utility values from CNR-ILS algorithm when the demand for nurses is three per shift and when the demand is four per shift. The difference is calculated as the value when demand is three nurses minus the value when demand is four nurses.

		Nurse Utility Difference ((CNR-ILS When Nursing Demand is 3)-(CNR-ILS))																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	0.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-2.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.70	3.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.20	0.00	0.49
	3	0.00	0.00	0.00	1.20	0.00	0.00	0.00	0.00	9.30	0.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00	0.00	0.00	4.30	0.80
	4	2.60	0.00	10.30	0.00	3.50	0.00	0.00	0.00	-0.70	0.00	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.00	1.50	0.91
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.60	0.00	0.70	0.00	0.00	6.30	0.00	0.00	0.00	0.00	0.00	0.48
	6	0.00	8.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.20	0.00	5.30	0.00	0.00	0.00	0.00	0.76
	7	7.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.30	1.30	1.30	0.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00	0.00	0.63
	8	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	2.50	0.17
	9	0.00	1.30	2.40	0.00	-1.20	0.00	1.20	8.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.80	0.00	0.00	0.00	0.00	0.81
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	4.50	0.00	9.30	0.00	0.71
	11	-2.40	3.10	0.00	0.00	0.00	0.00	0.00	0.00	5.00	1.20	13.00	0.00	0.00	1.20	0.00	1.29	2.50	0.00	11.50	0.00	1.82
	12	0.00	0.00	0.00	1.20	7.60	0.00	0.00	0.00	0.00	3.70	0.00	1.30	0.00	2.50	0.00	0.00	0.00	0.00	1.50	1.20	0.95
	13	0.00	0.00	2.50	0.00	0.00	2.50	0.00	0.00	0.50	11.50	5.10	31.80	0.00	1.20	0.00	0.00	0.00	0.00	46.50	0.00	5.08
	14	2.10	0.00	0.00	2.90	0.90	0.00	6.70	6.30	0.00	0.00	0.00	0.00	1.20	0.00	0.00	0.00	1.20	0.00	0.00	2.90	1.21
	15	0.00	2.50	10.20	0.00	0.00	1.30	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.00	1.30	3.50	0.00	1.54
	16	0.00	0.00	7.30	1.30	2.20	0.00	1.20	0.00	0.40	2.50	-1.30	13.90	3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.53
	17	4.80	0.00	0.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.20	0.00	4.10	0.00	0.00	0.00	0.92
	18	1.30	0.00	7.70	0.00	0.00	0.00	0.00	0.00	0.00	5.00	-1.20	0.00	0.00	0.00	0.00	0.00	2.50	0.00	0.00	0.00	0.77
	19	0.00	0.00	0.00	4.40	0.00	0.00	0.00	1.20	14.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02
	20	1.30	0.00	0.00	3.20	0.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00	3.40	3.80	1.30	-0.01	0.00	-0.40	0.00	0.00	0.78
AVG		0.85	0.91	2.02	0.77	0.65	0.36	0.56	0.73	1.69	1.26	0.86	2.39	0.38	0.50	0.84	1.08	0.74	0.14	4.03	0.62	1.07
STDV		2.08	2.08	3.65	1.30	1.90	0.87	1.55	2.40	3.87	2.92	3.12	7.58	1.00	1.03	2.24	2.54	1.45	0.42	10.61	1.24	3.60
Paired T		1.83	1.95	2.48	2.66	1.53	1.82	1.60	1.35	1.96	1.93	1.23	1.41	1.69	2.15	1.67	1.91	2.28	1.42	1.70	2.24	5.92
LCI		0.69	0.74	1.73	0.67	0.50	0.29	0.43	0.54	1.38	1.03	0.61	1.78	0.30	0.41	0.66	0.88	0.63	0.10	3.19	0.52	1.00
UCI		1.01	1.07	2.31	0.88	0.80	0.42	0.68	0.91	2.00	1.49	1.11	2.99	0.46	0.58	1.01	1.28	0.85	0.17	4.86	0.72	1.13

Table E-23: This table presents the difference between the nurse utility values from CNR algorithm when there are 20 nurses and when there are 16 nurses. The difference is calculated as the 20 nurse values minus the 16 nurse values. The 16 nurses that are compared were determined by deleting two nurses form both the night and day shift of the 20nurse CNR experiment.

		Nurse Utility Difference (CNR- (CNR when there is only 16 Nurses))																				
		Run																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AVG
Nurse	1	3.30	20.00	-1.80	24.70	9.60	6.80	17.80	-2.10	19.10	-26.40	4.30	-1.20	3.00	-0.90	-6.60	-15.20	0.00	-9.49	2.20	-24.20	1.15
	2																					######
	3																					######
	4	-3.00	75.60	-36.40	-30.00	-29.30	-12.50	35.50	11.00	0.30	0.50	14.70	-4.60	2.50	34.60	-25.30	30.50	21.20	21.90	31.30	30.70	8.46
	5	18.50	-3.50	-7.60	-1.30	-5.60	21.30	9.40	2.20	-3.90	-49.10	34.80	-8.80	0.00	-16.30	-1.20	1.20	-10.00	-1.40	1.30	7.80	-0.61
	6	3.20	-10.80	2.60	0.00	0.00	1.20	-1.30	14.90	54.00	8.10	8.20	4.80	-3.80	21.30	25.60	0.00	5.40	0.00	26.30	0.00	7.99
	7	-6.00	1.30	53.96	-10.30	0.00	-1.80	3.50	0.40	-1.20	1.40	-12.50	-4.40	-2.10	5.30	4.00	-3.70	63.10	60.40	56.20	-2.10	10.27
	8	8.60	40.00	0.00	9.30	5.00	68.30	8.70	25.80	-5.70	3.00	61.20	3.10	10.30	3.40	22.00	12.20	20.50	6.00	-4.70	18.80	15.79
	9	0.00	-1.20	31.00	-0.50	-11.30	3.40	-14.90	-17.10	4.40	-18.90	5.80	-0.50	52.90	-6.00	-8.70	41.90	29.80	15.10	0.00	-3.80	5.07
	10	10.60	30.00	-1.40	0.00	-0.80	7.30	6.60	15.80	-4.30	2.40	6.30	0.00	-12.70	7.00	-40.50	13.30	-5.00	1.20	-13.50	36.50	2.94
	11	-3.70	-68.70	-3.00	-3.00	15.00	3.20	36.40	57.60	27.50	-6.00	-19.80	5.00	-8.20	-2.90	-1.90	0.01	14.30	-39.40	-10.00	-1.30	-0.44
	12	63.60	11.20	66.90	7.40	-0.20	59.90	0.00	0.10	2.00	22.10	13.50	-5.60	27.40	46.80	31.30	25.30	4.20	67.30	29.10	71.66	27.20
	13																					######
	14																					######
	15	16.00	-13.00	-10.20	1.20	2.50	38.70	22.00	8.90	12.50	1.30	6.30	-6.00	-1.50	0.00	-16.50	-1.80	-1.50	17.80	-7.00	31.50	5.06
	16	-3.00	10.80	40.80	0.60	17.80	10.60	20.00	25.60	56.30	16.30	-13.10	62.50	-1.30	68.00	88.30	18.00	-15.10	17.50	38.10	-4.10	22.73
	17	-9.50	0.80	6.60	1.20	0.70	4.60	0.00	0.00	-6.10	0.00	0.00	-20.30	-1.20	-0.50	42.90	-2.50	13.80	9.70	-32.40	-2.70	0.26
	18	-2.30	7.00	-2.30	9.00	45.20	-0.90	20.00	2.10	11.90	11.70	11.10	-3.10	11.80	56.70	20.00	0.00	-12.10	-20.20	28.80	20.00	10.72
	19	-1.20	-16.20	-1.20	-1.60	1.30	23.60	1.20	0.00	-14.00	0.10	-5.40	0.00	0.00	-1.70	2.50	0.00	-1.20	0.00	1.50	11.70	-0.03
	20	11.30	18.00	-5.40	58.40	25.50	-4.70	31.30	29.80	23.10	79.70	-8.80	13.80	0.00	-2.50	1.20	24.20	64.20	2.60	40.50	22.30	21.23
AVG		6.65	6.33	8.29	4.07	4.71	14.31	12.26	10.94	10.99	2.89	6.66	2.17	4.82	13.27	8.57	8.96	11.98	9.31	11.73	13.30	8.61
STDV		17.20	30.41	26.46	18.26	16.38	22.98	14.62	17.49	20.70	26.70	19.65	17.68	15.69	24.86	30.20	15.34	23.82	26.22	24.23	22.59	21.74
Paired T		1.55	0.83	1.25	0.89	1.15	2.49	3.36	2.50	2.12	0.43	1.36	0.49	1.23	2.13	1.13	2.34	2.01	1.42	1.94	2.36	7.08
LCI		5.13	3.64	5.94	2.45	3.26	12.28	10.97	9.39	9.16	0.52	4.92	0.60	3.43	11.07	5.90	7.60	9.87	6.99	9.59	11.30	8.18
UCI		8.17	9.02	10.63	5.68	6.16	16.35	13.56	12.49	12.83	5.25	8.40	3.73	6.21	15.47	11.24	10.32	14.08	11.63	13.88	15.30	9.04

Table E-24: This table presents the results from test for equality of means between the IP, CNR, and CNR-ILS models. CNR-ILS 4 = CNR-ILS where demand for nurses is four per shift; CNR-ILS 3 = CNR-ILS where demand for nurses is three per shift; ADV CNR = CNR with the AT Mechanism; ADV CNR-ILS = CNR-ILS with the AT mechanism.

NURSE UTILITY VALUES	
AVG PER RUN	
	TTEST
IP to CNR	**0.06**
IP to CNR ILS	**0.03**
CNR-ILS 3 to CNR ILS 4	**0.29**
CNR to CNR ILS	**0.000635**
CNR to ADV CNR	**0.178281**
CNR-ILS to ADV CNR ILS	**0.398896**
OVER ALL NURSES AND ALL RUNS	
	TTEST
IP to CNR	**0.05**
IP to CNR ILS	**0.02**
CNR-ILS 3 to CNR ILS 4	**0.27**
CNR to CNR ILS	**0.00**
CNR to ADV CNR	**0.18**
CNR-ILS to ADV CNR ILS	**0.39**

RATIO OF ROS SATISFIED	
AVG PER RUN	
	TTEST
IP to CNR	**0.00**
IP to CNR ILS	**0.06**
CNR-ILS 3 to CNR ILS 4	**0.09**
CNR to CNR ILS	**0.000000**
CNR to ADV CNR	**0.000047**
CNR-ILS to ADV CNR ILS	**0.265442**
OVER ALL NURSES AND ALL RUNS	
	TTEST
IP to CNR	**0.00**
IP to CNR ILS	**0.04**
CNR-ILS 3 to CNR ILS 4	**0.08**
CNR to CNR ILS	**0.00**
CNR to ADV CNR	**0.00**
CNR-ILS to ADV CNR ILS	**0.25**

SINGLE DAYS OFF	
AVG PER RUN	
	TTEST
IP to CNR	**0.02**
IP to CNR ILS	**0.29**
CNR-ILS 3 to CNR ILS 4	**0.36**
CNR to CNR ILS	**0.074172**
CNR to ADV CNR	**0.100531**
CNR-ILS to ADV CNR ILS	**0.113697**
OVER ALL NURSES AND ALL RUNS	
	TTEST
IP to CNR	**0.01**
IP to CNR ILS	**0.28**
CNR-ILS 3 to CNR ILS 4	**0.35**
CNR to CNR ILS	**0.05**
CNR to ADV CNR	**0.11**
CNR-ILS to ADV CNR ILS	**0.10**

Table E-25: This table presents the survey results from Mike O'Callaghan Federal Hospital.

NURSE SATISFACTION RATINGS																											
CNR W/O DEMAND																											
Run	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**AVG**	**STD**	**TOTAL AVG**	**TOTAL STD**	**BAD RESPONSE**	**RESPONSES**	**% BAD RESP**
	-5	-5	2	5	-2	2	2	1	2	5											**0.7**	3.592	1.229166667	3.026405658	3	10	0.3
	2	-1	-1	-3	-1	2	-1	-3	5	0	5	-1	5	2							**0.71429**	2.813			7	14	0.5
	-1	5	2	-5	4	4	2	4	4	5	5										**2.63636**	3.107			2	11	0.181818182
	2	1	2	4	4	1	-1	-3	5	2	-4	1	-1								**1**	2.677			4	13	0.307692308
																									16	48	0.333333333
CNR W/ DEMAND																											
Run	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**AVG**	**STD**	**TOTAL AVG**	**TOTAL STD**	**BAD RESPONSE**	**RESPONSES**	**% BAD RESP**
	5	3	3	**4**	2	3.5	3	4	2	5											**3.45**	1.066	2.760416667	1.943318021	0	10	0
	3	-1	-1	2	4	3	3	3	**3**	3	5	0	5	3							**2.5**	1.912			2	14	0.142857143
	4	5	5	4	**-1**	4	3	**3**	4	5	**0**										**3.27273**	2.005			1	11	0.090909091
	-1	5	**1**	**3**	4	2	1	-3	5	2	4	1	3								**2.07692**	2.326			2	13	0.153846154
																									5	48	0.104166667
IP MODEL																											
Run	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**AVG**	**STD**	**TOTAL AVG**	**TOTAL STD**	**BAD RESPONSE**	**RESPONSES**	**% BAD RESP**
	-3	4	2	-5	1	4	5	4	1	4											**1.7**	3.335	1.583333333	2.75860999	2	10	0.2
	4	1	3	-2	5	5	0	3	-1	5	-3	-1	4	2							**1.78571**	2.778			4	14	0.285714286
	5	2	2	-1	2	3	2	0	3	0	0										**1.63636**	1.748			1	11	0.090909091
	4	5	3	-3	4	1	2	0	5	2	-5	1	-3								**1.23077**	3.219			3	13	0.230769231
																									10	48	0.208333333
MOFH SCHEDULE																											
Run	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**AVG**	**STD**	**TOTAL AVG**	**TOTAL STD**	**BAD RESPONSE**	**RESPONSES**	**% BAD RESP**
	5	3	-1	-4	5	3	4	5	4	5											**2.9**	3.035	1.729166667	2.765590879	2	10	0.2
	-1	4	5	-3	3	1	2	-3	-1	0	5	3	2	2							**1.35714**	2.649			4	14	0.285714286
	2	1	5	-1	-2	3	0	2	4	-3	0										**1**	2.49			3	11	0.272727273
	-2	5	2	5	4	1	3	4	5	0	-4	2	-1								**1.84615**	2.911			3	13	0.230769231
																									12	48	0.25

Table E-26: This table presents the number of LRB iterations that were required to solve disruptions that entered the IM phase of the CNRR algorithm during rerostering.

REQUIRED LRB ITERATIONS								
Day=0						Day=14		
Can't Change Ros			Can Change Ros			One week Can't Change ROs		
1-day	2-day	3-day	1-day	2-day	3-day	1-day	2-day	3-day
1	1	121	1	1	121	1	1	126
1	1	121	1	1	121	1	1	304
1	26	221	1	1	126	26	24	826
1	26	281	1	26	221	26	26	
1	26	607	1	26	249	26	291	
1	117		1	26	281	112	291	
1	117		1	26		151	301	
1	191		1	116			301	
1	191		14	116			301	
14	212		22	191			301	
24	237		24	212			496	
26	274		26	214			826	
26			26	226			901	
49			49	316				
86			86	621				
281			236					
			281					
32.1875	118.25	270.2	45.41176	141.2667	186.5	49	312.3846	418.6667
70.38676	100.3459	200.3078	83.67053	167.773	72.48103	58.54912	289.3267	363.815